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Instructor's Handbook on Meteorological Instrumentation

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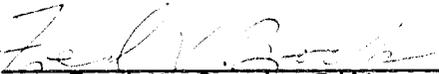
PREFACE

The objective in preparing this Instructor's Handbook is to provide useful material to assist instructors in teaching meteorological instrumentation. It is intended to provide flexibility in meeting the needs of individual teaching programs. The Handbook, therefore, provides a variety of teaching materials: text, references, questions, problems, and laboratory exercises. Its purpose is to fill the existing gap in meteorological teaching texts, until better textbooks become available.

An instructor using this Handbook faces two problems: 1) how to get the material to the students, and 2) how to adapt it to a specific program. These problems were taken into consideration and solved by not copyrighting the Handbook and producing it in a loose leaf, three-hole-punched form. It may be personally copied and distributed for only the cost of reproduction. The instructors may either delete or add relevant material to obtain their personal program plan.

The history of the ATD Colloquium on the Teaching of Meteorological Instrumentation is exemplified in Chapter 1 by James F. Kimpel. The colloquium received the full endorsement of the University Corporation of Atmospheric Research (UCAR), the National Center for Atmospheric Research (NCAR), the National Science Foundation (NSF), and the Board of Meteorological and Oceanographic Education in the Universities of the American Meteorological Society. In behalf of the participants, I would like to thank Wilmot Hess, Director of NCAR, and Robert Serafin, Director of NCAR's Atmospheric Technology Division, for their assistance and support. I would also like to thank the participating universities for providing support to their faculty members who attended. In addition, the contributors deserve the highest praise for their diligence and their very fine contributions.

Special thanks go to the following NCAR staff. Harold Cole and Julian Pike assisted greatly in the preparation and presentation of lectures for the colloquium. Tony Darnell assisted in preparing materials. Diane Wilson made all local arrangements. The assistant Editor, Carol Nicolaidis, coordinated and supervised the process of getting the diverse contributions into a coherent form. This was a difficult and challenging task and she did a splendid job.


Fred V. Brock, Editor

30 May 1984

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CHAPTER 1

INTRODUCTION

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1.0 STATEMENT OF THE PROBLEM

The meteorology/atmospheric science community has expressed a growing concern in recent years over the status of education and research on instrumentation and measurement systems. This concern is most evident in the vast range of subject material and emphasis in meteorological course offerings taught in colleges and universities throughout the United States. Informal surveys have revealed that instrumentation/measurements is no longer required or even offered to students in many undergraduate degree programs. In some instances meteorological analysis, statistics, or computer programming is taught under the guise of a course with an instrumentation/measurements title. Few courses exist in this subject area at the graduate level. In most cases the graduate student's exposure to formal coursework in instrumentation/measurements occurs piecemeal as a portion of other courses such as cloud physics, radiation, turbulence and diffusion, etc.

This situation is exacerbated by the fact that no formal text in meteorological instrumentation/measurements is currently in print. In addition, the void created by the demise of the National Science Foundation's (NSF) Science Education programs has not been filled in most institutions by increased priorities on developing and equipping instructional instrumentation laboratories. This unfortunately occurs at a time when improved technologies are available in sensor design and when computer-based data handling and display systems are becoming relatively inexpensive.

A continuation of this situation has far reaching implications. Meteorology graduates will not be qualified to select the instrumentation and collect measurements necessary to undertake field studies or to make contributions toward improving the synoptic data gathering networks. Valuable employment opportunities will quite possibly be filled with graduates from other disciplines. In other areas of the science, a lack of exposure to instrumentation/measurements will contribute toward the growing problem of meteorological data being often misinterpreted and misused.

1.1 BACKGROUND

The problems discussed above were subjects of lively debate in small informal groups prior to the summer of 1982. It was approximately then that individuals at the National Center for Atmospheric Research (NCAR) and members of the American Meteorological Society's Board of Meteorological Education in Universities (AMS-BMEU) jointly proposed that the situation be formally discussed and that a plan for action be developed.

In October 1982, Bob Serafin and Fred Brock of NCAR visited with approximately 45 individuals participating in the Third Meeting of the Heads and Chairs of Departments of Atmospheric Science. Serafin and Brock asked if there was general interest in a NCAR-sponsored summer colloquium on meteorological instrumentation/measurements. The response of the department chairs or their representatives was enthusiastic and encouraged both NCAR and the AMS-BMEU to proceed toward a colloquium in the summer of 1983 (Kimpel, 1983).

The topic was subsequently discussed at the December 1982 meeting of the University Corporation for Atmospheric Research (UCAR) University Relations Committee. There an alternate plan was proposed. The alternate plan involved NCAR's developing a travelling instrument/measurements laboratory that would visit universities periodically as part of a locally taught course or seminar. The main objection to this plan was one of cost and that only a few universities could be visited each year. The summer colloquium plan was endorsed along with the idea that the participating universities could possibly share in the cost. The possibility of designing the colloquium primarily for faculty was also discussed.

The AMS-BMEU met in January 1983 in conjunction with the 63rd Annual Meeting of the AMS. Bob Serafin opened the discussion of a meteorological instrumentation colloquium, reviewed progress to date, and requested advice from the Board. The Board also heard results of a pilot program conducted at Iowa State University (ISU) by Fred Brock and Gene Takle (ISU). Takle indicated that the pilot program was extremely worthwhile and that much could be gained from the exposure of university faculty and students to the instrumentation and expertise available at NCAR.

The concept of a colloquium received encouraging support. The AMS-BMEU then made several suggestions:

1. The colloquium should be offered at NCAR.
2. The colloquium on instrumentation/measurements should be involved in instrumentation instruction. This would ensure that a large number of students, both undergraduate and graduate, could directly benefit through improved course materials.

3. The colloquium should produce a written document so that the materials discussed would be made available to individuals from colleges and universities not participating in the colloquium.

4. The subject matter of the colloquium should focus on standard meteorological instrumentation and measurement systems. Exotic remote sensing and experimental observation systems were regarded as being beyond the scope of the colloquium.

5. The cost of the colloquium should be shared between NCAR and the participating universities. Perhaps the universities could pay the salaries of the participants while NCAR covered the costs of attending the colloquium.

The above-mentioned discussions were taken under advisement by NCAR, and the Atmospheric Technology Division (ATD) announced in February 1983 that it would sponsor the Colloquium on the Teaching of Meteorological Instrumentation, July 25 through August 5, 1983, in Boulder, Colorado.

1.2 THE 1983 ATD COLLOQUIUM ON THE TEACHING OF METEOROLOGICAL INSTRUMENTATION

The colloquium attracted 23 participants including 14 faculty members and two senior graduate students from the UCAR universities, four faculty members from non-UCAR universities, and three scientists from other organizations. Principal instructors were Fred Brock, Scientist, and Julian Pike, Engineer, of NCAR's Field Observing Facility, and Harold Cole, Engineer, of NCAR's Global Atmospheric Measurements Program. Guest lectures were given by Ron Rinehart, Convective Storms Division, Ed Flowers and Chandran Kaimal, National Oceanic and Atmospheric Administration (NOAA). Fred Brock also served as colloquium coordinator.

Initial discussions focused on the purpose and objectives of the colloquium. It was generally agreed that a systems approach to meteorological measurements would be followed for immersion sensors, including the supporting components that provide coupling to the atmosphere, data acquisition, communications, display, and processing.

The group quickly organized to develop a written record of the workshop. It was decided that colloquium proceedings should take the form of an instructor's manual, that is, a document that could serve as a basis for an up-to-date course in meteorological measurements.

1.3 ORGANIZATION OF THE INSTRUCTOR'S MANUAL

This manual is intended to assist instructors in teaching undergraduate courses in meteorological instrumentation and measurements. It may also be useful background to those teaching other courses requiring the use of meteorological data, analysis, display, and/or processing. It was written with the assumption that the potential student audience is familiar with mathematics through differential equations, calculus-based physics, general meteorology, and computer programming. These basic concepts are not reviewed in this manual. However, background prerequisites are often listed at the beginning of each section along with appropriate references. In those instances where a meteorological measurements course is taught at the sophomore or junior undergraduate level, the instructor may find it necessary to condense and simplify portions of this manual for lectures and laboratory presentations. On the other hand, instructors of advanced graduate courses will find it necessary to add supplementary material from their own expertise and experience.

The manual is organized as closely as possible to follow a typical meteorological measurement system such as described in Doebelin (1975). Chapter 2 discusses the performance characteristics of sensors in general. Chapter 3 reviews basic calibration standards. Chapter 4 discusses sensors and transducers commonly used to obtain environmental measurements of temperature, humidity, wind, pressure, solar and terrestrial radiation, precipitation, and evaporation. Chapter 5 discusses the field environment with regard to exposure, platforms, maintenance, and data quality assurance. Chapter 6 develops the concept of a complete measurement system from a simple sensor and visual indicator through complex sensors, automated analog to digital conversion, and recording assemblages. Chapter 7 deals with digital data processing to achieve high information content. Examples of questions, problems, class demonstrations and laboratories have been included, where possible, at the end of each major section. A list of recommended minimal equipment to support a small enrollment undergraduate course in meteorological measurements is included as an appendix.

It is the hope of the participants and instructors at the 1983 ATD Colloquium on the Teaching of Meteorological Instrumentation that this instructor's manual will serve to fill a critical need until a suitable text and laboratory manual becomes available.

1.4 REFERENCES

Doebelin, E.O., 1975: Measurement Systems: Application and Design.
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Kimpel, J.F., 1983: Third Meeting of the Heads and Chairmen of
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Soc., 64, 786-791.

CHAPTER 2

PERFORMANCE CHARACTERISTICS OF SENSORS

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2.0 INTRODUCTION

2.0.1 PERFORMANCE CHARACTERISTICS: INPUT/OUTPUT (I/O)

The performance characteristics of a measuring system provide a quantitative description of how well the system measures the primary inputs and indicate how it responds to spurious inputs. Careful specification of desired characteristics is essential during the design phase of any new system. Also, when attempting to identify a system suitable for a proposed measurement, critical evaluation of performance parameters is required. Consequently, individuals working with measurement systems should have a clear understanding of the various performance characteristics.

2.0.2 STATIC VS DYNAMIC CHARACTERISTICS

The quality of a measurement of a physical quantity (temperature, wind speed, etc.) can be assessed by defining a set of performance characteristics for the sensor measuring that particular quantity. There are two types: static performance characteristics and dynamic performance characteristics. The static performance characteristics of a sensor can be defined as the observed output of the sensor. The dynamic performance characteristics of a sensor define how a sensor will respond to a time-varying physical quantity (input).

The key word in delineating between static and dynamic characteristics is time. The quality of a measurement is influenced by both characteristics as nonlinearities. Furthermore, the approximation is generally made that static characteristics of a sensor are assessed by excluding dynamic effects, and the dynamic characteristics are assessed assuming that there are no static errors. These assumptions appear when describing first- and second-order systems as applied to meteorological sensors.

2.0.3 OBJECTIVE OF CHAPTER

The instructor who is to teach meteorological instruments to upper-division, undergraduate majors in meteorology may realize that the specifications of commercially available instruments are most often insufficient to judge the performance characteristics of these instruments. He might decide, therefore, to develop the principles and criteria of performance for classroom presentation and laboratory application. It is the purpose of this chapter to present the basic concepts of the performance characteristics of meteorological instruments. Recent developments in the field of meteorological instrumentation and measurement have made it necessary to rethink the subject of performance, such that those scientists who wish to measure atmospheric conditions and processes will find basic guidelines.

Section 2.1 presents the treatment of the static characteristics of instruments. It emphasizes the response of a meteorological instrument to a time-independent input from the environment. A schematic I/O diagram is used to clarify some of the concepts which characterize instrumental performance. Subsequently, a series of definitions is given to provide deeper insight into the broad diversity of modern considerations surrounding the subject of static characteristics. Pertinent questions, problems, and exercises, which the instructor may find useful in structuring lab and lecture activities, are provided in Section 2.1.5.

Section 2.2 addresses the problem of dynamic characteristics. First, the physical principles underlying the mathematical model equations are discussed, such that the student may recognize where these equations come from. In this context, the energy reservoir concept is explained, which may be considered a criterion for judging the order of an instrumental system. Particular emphasis is given to the first- and second-order systems and the respective equations. Methods of solution are discussed in such a way that the student will recognize the significance and limitations of specific solution forms. Various meteorological examples further serve this purpose. A brief discussion of higher order and nonlinear instrumental systems develops the broader scope within which first- and second-order models function. Again, appropriate questions, problems, and lab exercises provided in Section 2.2.6 are intended to facilitate increased conceptual grasp and applicability of the models.

2.0.4 COMPLEXITY OF DEFINING TERMS

One difficulty encountered in discussing performance characteristics of instruments is the considerable variation in the way terms are defined in the literature.

In many cases authors discuss the same concepts but have semantic differences in referring to these concepts. It is more important for

students (and instructors) to understand the concepts rather than the exact terms. However, various definitions often lead to difficulties in discussing instrument behavior, and it is especially important to be sure of the definitions being used when reading manufacturer's specifications.

The order of presentation also varies between authors. We have postponed the discussion of accuracy and precision until all the other static characteristics have been discussed so as to be able to refer to various forms and sources of imprecision and inaccuracy.

2.1 STATIC CHARACTERISTICS

2.1.1 DETAILED DEFINITION OF A STATIC CHARACTERISTIC

The static performance characteristics aid in defining how well a sensor will respond to the input variables that effect the response of the sensor. For example, if a pressure sensor's principal input is pressure, its secondary input would be temperature because of the temperature dependency most pressure sensors exhibit. All input variables other than the primary ones should be minimized and quantified.

A static calibration is performed to assess the static characteristics for the sensors. The static calibration process requires that either the primary or the other input variables are varied, while the remaining variables are held constant. The actual calibration is accomplished by applying many inputs over the range of the sensor and examining the functional relationship between the known input and the exhibited output in physical units. The relationship between the output and the known input after the sensor has reached equilibrium will determine the static performance characteristics. The set of characteristics used to describe the sensor sensitivity to the input variables is best described using an I/O diagram.

2.1.2 CONCEPT OF AN I/O DIAGRAM

The primary purpose of an I/O diagram is to display a functional relationship between a known input (ultimately, an atmospheric signal) into the measuring system and the output of the system. Ideally, the inputs used to determine the I/O diagram would be actual atmospheric signals, but they are generally too complex both spatially and temporally. Consequently, the relationship between the input and the output is normally established during a laboratory calibration process. By applying a series of carefully generated input values and noting the corresponding outputs, a number of data points (I/O pairs) can be obtained. Using an appropriate least-squares technique, a "best-fit"

expression to these data points can be obtained. This expression is then used as the I/O relationship for the system. It is best displayed as a plot on rectangular coordinate paper (see Fig. 2-1 for a schematic example). The abscissa should be the input, and the ordinate should be the output in units appropriate to the actual physical output, not the meaning attached to any scale markings. For example, a system with a dial type output should have the system output plotted in units of angular degrees of rotation of the pointer, not in terms of the units of the dial scale. Following this convention facilitates comparison of one system with another in regards to ability to detect input changes.

Once constructed, the I/O diagram is a valuable tool in assessing system performance. The next subsection uses this diagram in defining various terms commonly used to describe the static performance of measuring systems.

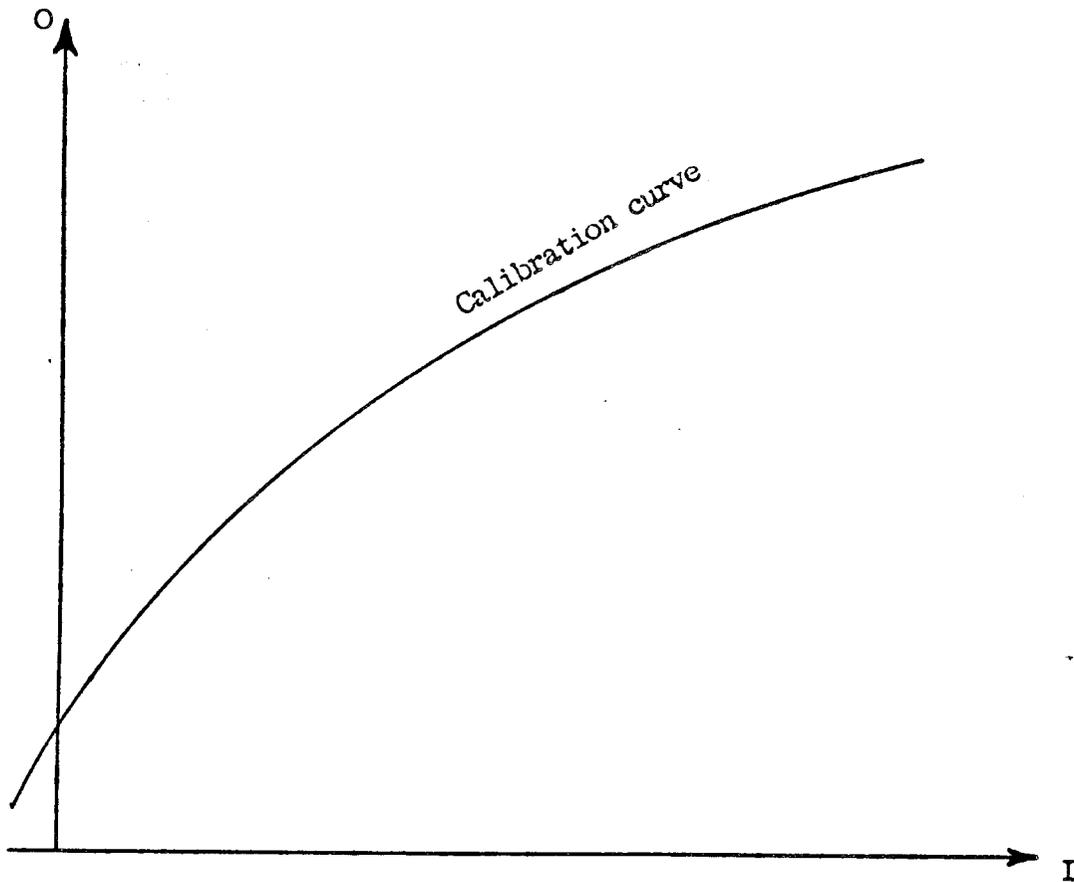


Fig. 2-1. Schematic diagram of the I/O relationship.

2.1.3 DEFINITION OF TERMS

In studying the static performance characteristics of sensors, one finds it useful to base the treatment on a series of fundamental concepts. Some authors offer formal definitions for these concepts, e.g., Bottaccini, 1975, Chap. 1. However, the present discussion rests on the I/O diagram as presented in the previous section. For a specific instrument, such a diagram is found by means of a calibration process under controlled conditions in the laboratory. All interfering (secondary) inputs are held to a minimum; whereas, the primary input of interest is changed in a time-independent manner.

Sensitivity. The slope to the I/O curve represents the sensitivity of an instrument. If this curve is not a straight line but rather a non-linear curve, the value of the slope depends on the chosen value of the input (Doebelin, 1975, Chap. 1, p. 7). Often, for commercial instruments an averaged linear I/O relationship is offered as a statistical best-fit. It is reemphasized that the sensitivity describes a time-independent condition (static characteristic).

Zero offset or bias. The I/O curve need not go through zero (origin of the diagram). Instead, it may cross the output axis at some positive or negative value. This ordinate value specifies the zero offset or bias. This parameter is also called the systematic error (Doebelin, 1975, Chap. 3, p. 58), to which the term systematic error is related (Bottaccini, 1975, Chap. 1, p. 7).

Range. A specific type of instrument is designed to be responsive within a given interval of the monitored quantity: the range. For example, a thermometer may have a range from -40°C . to $+60^{\circ}\text{C}$.

Span. The algebraic difference between the upper and the lower values which specify the range of a measuring instrument is designated its span. In the example just given, the span is 100°C .

Resolution. The resolution of an instrument is determined by the smallest variation in the environmental input variable that causes a detectable change in the instrumental output. Gill and Hexter (1972, p. 848) explain the term resolution by referring to a resistance thermometer with a span of 100°C and a 1000-turn slide wire. A new and appropriately calibrated instrument may have a resolution of $+0.1^{\circ}\text{C}$ in correspondence with one turn of the slide wire. However, if the sliding contact is worn, the resolution could be as bad as $+1.0^{\circ}\text{C}$ or worse.

Whereas the sensitivity of an instrument as related to its primary input is a fundamental concern, its sensitivity to interfering secondary inputs is not always negligible. In considering temperature as a spurious input into a pressure gauge, one recognizes thermal expansion or contraction that may vary the output reading with pressure being unchanged. Variations due to temperature of the elastic properties of the gauge will also generate a change in the pressure

sensitivity. This condition would represent a modifying input, whereas the former effect causes an interfering input. The respective drifts are a zero drift and a sensitivity or scale-factor drift. A plot may make this clear.

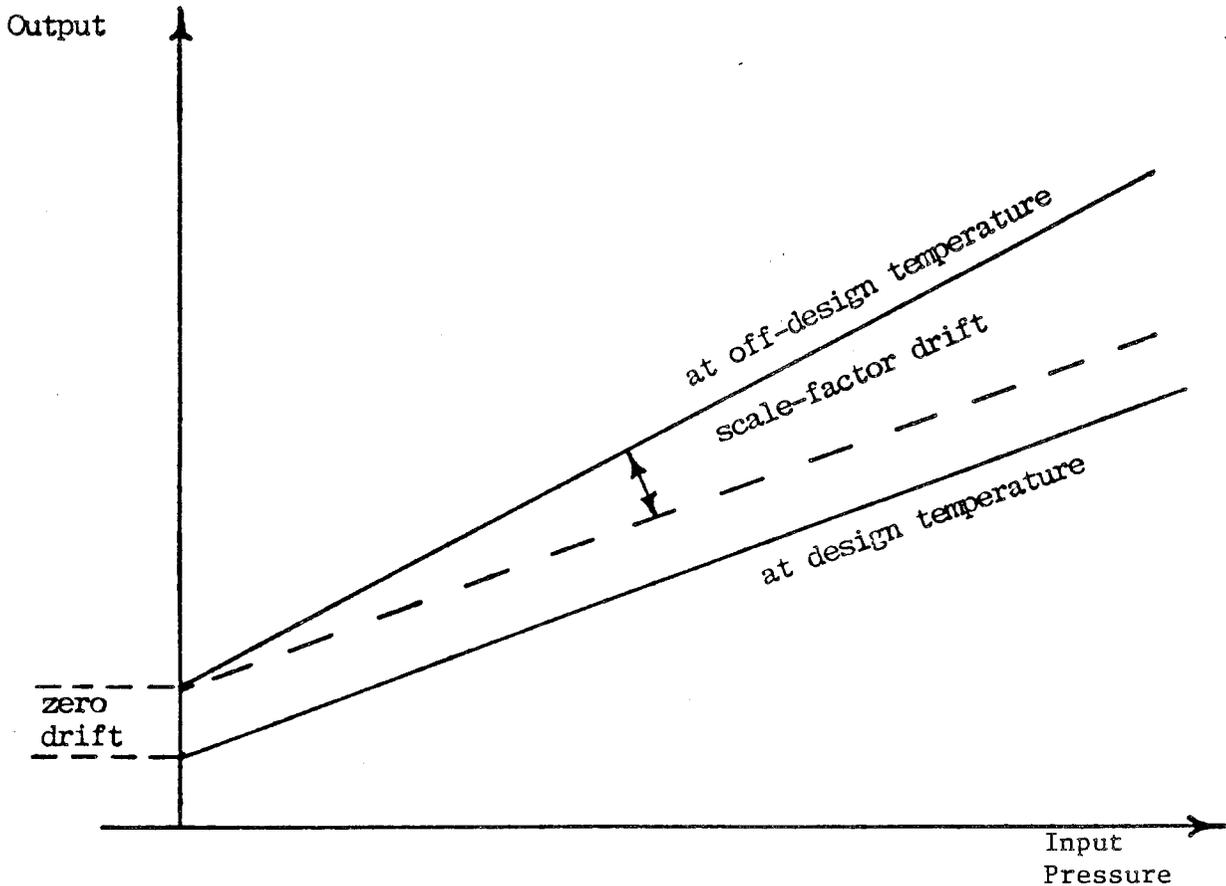


Fig. 2-2. Scale-factor and zero drift as effects of temperature dependence.

The zero drift causes a parallel displacement of the I/O curve. The scale-factor drift changes its slope at off-design temperatures.

Linearity. If the I/O curve of an instrument is nonlinear, the device may still be sufficiently accurate, once it is known what the measure of nonlinearity is for the range of the instrument. Such a measure is often based on a reference straight line of least-squares fit. It is customary to express this measure, which is called linearity, in terms of the maximum deviation from the reference line divided by the full-scale range. Thus, linearity is a percentage measure of nonlinearity (Gill and Hexter, 1972, item 18, p. 850; Bottaccini, 1975, Chap. 3, p. 65). For an easier perception of the notion of linearity, see Fig. 2-3.

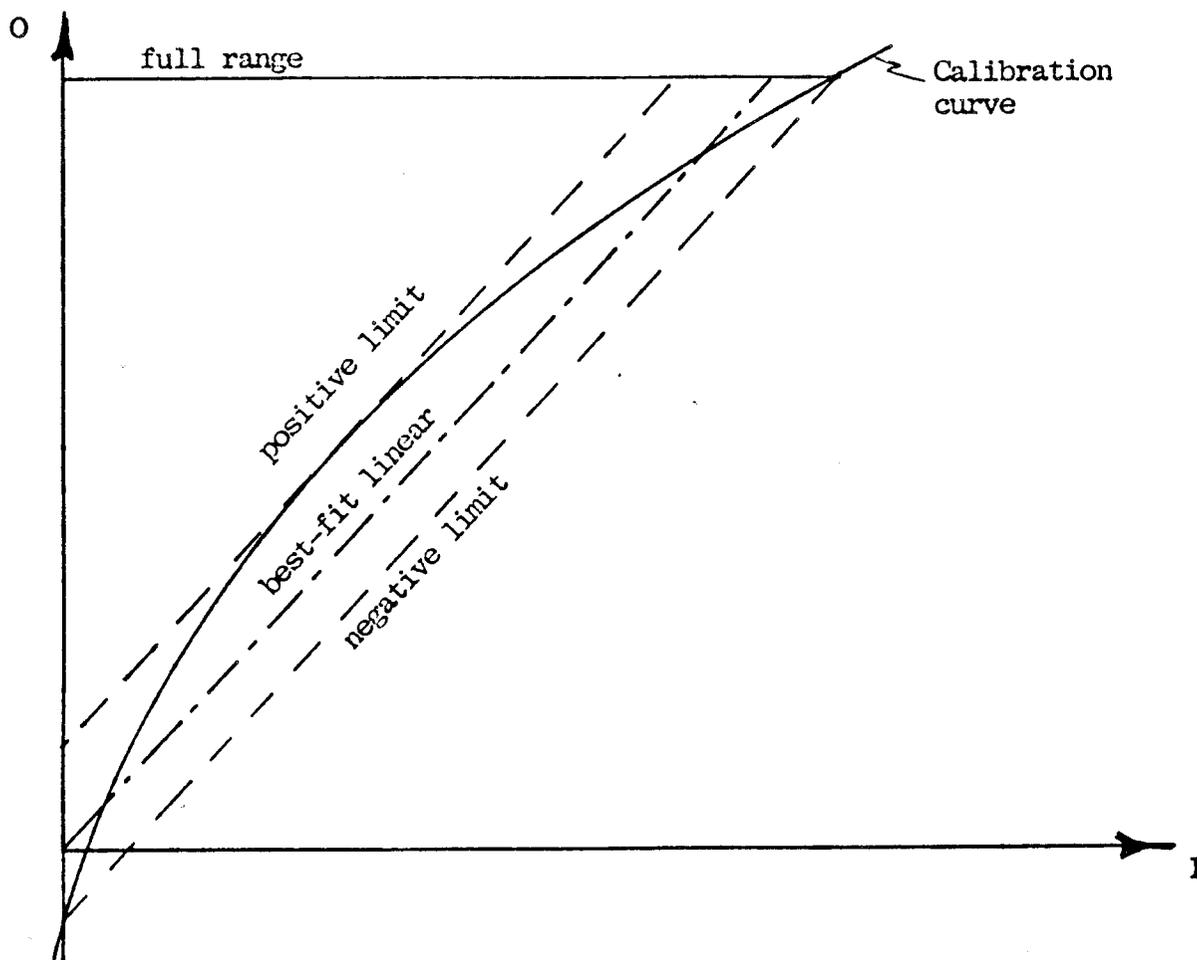


Fig. 2-3. Measure of linearity based on a nonlinear calibration curve.

Hysteresis. For any given input, a measuring instrument may generate a different output, depending on whether the specific input value is reached by an increasing or decreasing change of that quantity. This phenomenon is known as hysteresis. It is determined as the maximum difference in output when any input value is approached first with increasing and then with decreasing input signals (Gill and Hexter, 1972, item 12, p. 848). The cause for this hysteresis behavior is found in the energy absorption by the elements of the measuring instrument. Examples are some aneroid barometers and humidity elements in radiosondes.

Repeatability. If one considers the degree of proximity among consecutive measurements of the output for the same input value with no modification in the operating conditions, one deals with repeatability, which is actually quantified as the nonrepeatability in percent of span. In approaching the given input value from the same direction, one excludes hysteresis (Doebelin, 1975, Chap. 2, p. 73; Gill and Hexter, 1972, item 12, p. 848).

Long-term stability. A reliable instrument should not only be well calibrated but it is expected to hold its performance close to its calibration curve over a reasonable period of time. In addition, a reliable instrument should preferably settle over several calibrations toward its own ultimate state of performance (Bottaccini, 1975, pp. 10 and 65). Such a behavior is called long-term stability.

Threshold. In the definition above, resolution was given as the smallest measurable input change. If a measurement is started at zero input with the input values gradually increasing, the instrument may not respond to very small values. The smallest measurable input is named the threshold. Some manufacturers of commercial instruments state instead a kind of "threshold," which marks with decreasing values the input level when the instrument output drops to zero. As an example, consider the rotating cup anemometer. Due to static friction, the anemometer may not start rotating if wind speed gradually increases from calm conditions. Conversely, if environmental wind speed diminishes toward zero, dynamic friction would allow a lower value at which rotation ceases.

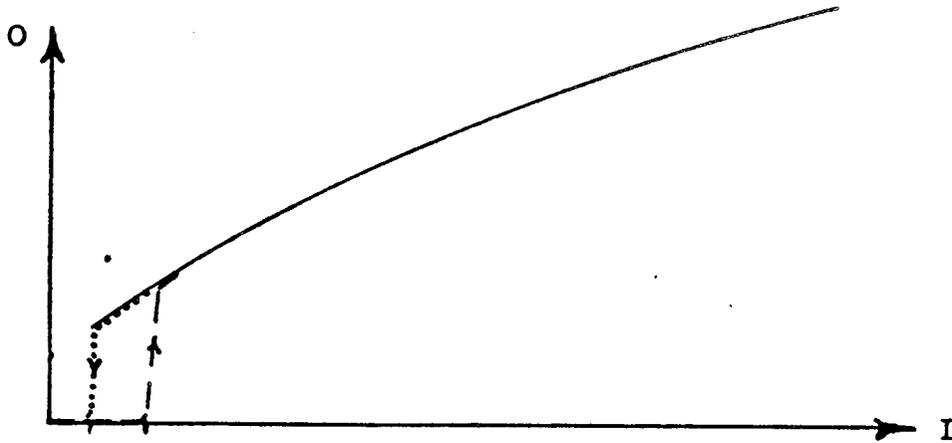


Fig. 2-4. Starting and stopping thresholds.

Dead-band. Practically all instruments demonstrate some lag of response. There exists a subrange through which the input varies without initiating a response. This range is defined as the dead-band, which is given in percent of full-scale range.

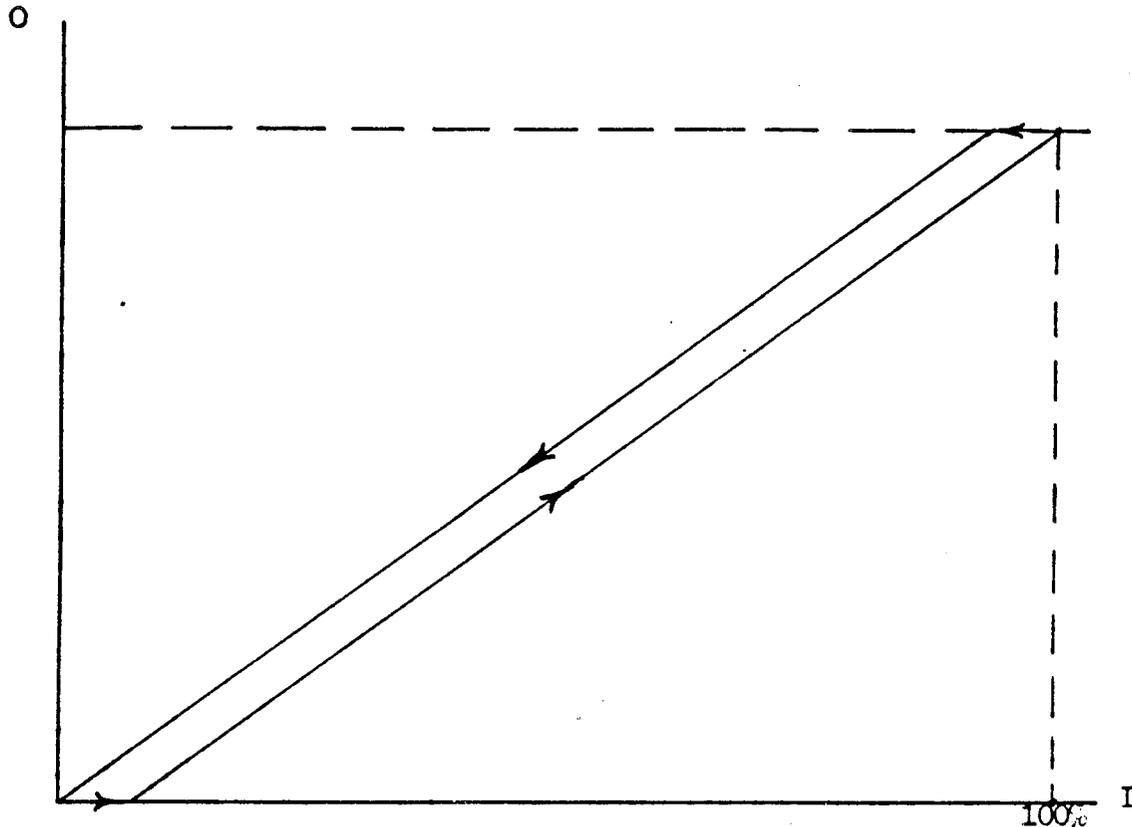


Fig. 2-5. The dead-band at 100% input change.

The plot in Fig. 2-5 may clarify the conditions of dead-band.

Precision. An instrument that generates minimal deviations from the most likely value for the measured quantities is a precise instrument. Bottaccini (1975, Chap. 1, pp. 6 and 32) offers a mathematical expression of precision. Determination of the most likely value of the output involves some statistics. The method of least squares is an often used procedure to arrive at this value. Lack of precision is caused by random error.

Accuracy. Although a principal interest exists in the accuracy of instruments, the concept of accuracy is rather complex. Recent development in instrumental design and application has stepped away from this notion as a practical term to describe the static performance characteristics. If one attempts a definition, one may state that the accuracy of an instrument is given by the degree with which it measures a specific environmental variable in terms of its true (yet never knowable) value. Bottaccini (1975, Chap. 1, p. 5) presents a formal

mathematical discussion of this concept. Doebelin (1975, Chap. 3, pp 44-58) treats accuracy in conventional detail. Some standard statistical development has been included. Gill and Hexter (1972, item 15, p. 849) points out that accuracy may include hysteresis, repeatability, and other errors.

2.1.4 COMMENTS ON THE RELATIVE IMPORTANCE OF STATIC RESPONSE CHARACTERISTICS

Upon examining the various static response characteristics, it becomes evident that not all of the characteristics are of equal significance or importance. Sensitivity, range, span, and resolution are properties exhibited by even the most ideal instruments. An instrument must be chosen that has values of these properties suitable for a particular scientific use. Properties such as linearity and no zero-offset are often desirable because they simplify use of the instrument; however, if the instrument is carefully calibrated and is to be used with a digital computer, computations may be done to correct for these properties. Note, however, that it is important to obtain linearity of a quantity before averaging or other linear processing is done on that quantity. Avoiding temperature dependence of various instrument performance parameters can also simplify use of an instrument, but corrections can be made with careful calibration, if the temperature of the instrument is measured. Hysteresis, drift, threshold, and dead-band are much more serious. These represent sources of imprecision and inaccuracy which generally cannot be corrected.

2.1.5 QUESTIONS, PROBLEMS, AND LABORATORY EXERCISES

Questions and Problems

1. Is it possible for a device to be very precise but not be very accurate? To be very accurate but not very precise? Can you think of some applications for which a meteorologist would choose an instrument which has a high precision even though it is less accurate than another instrument?
2. Is it possible for an instrument to be "too accurate?" Is accuracy per se ever a disadvantage in an instrument? If not, why don't we always use the most accurate instrument available?
3. Let's say that you are designing an experiment in which you will place microbarographs at various locations separated by 10 km and that the accuracy is ± 0.5 mb. To what accuracy will you be able to measure the geostrophic wind (speed and direction)?

4. Compare the readings shown by various thermometers at about the same time. One possible choice would be a thermometer in the meteorology building and thermometers located on several banks in tow. Read the various thermometers within as short a time as possible and make a second reading from the first thermometer which you used after you have completed all of the other readings. How much of the variation do you think is "real" (i.e., is due to actual differences in the temperatures at the various banks), and how much is due to instrumental inaccuracies? How accurate do you think the thermometers are?

Laboratory Exercises

Laboratory Exercise #1: The accuracy of reading an analog device.

Equipment:

(See below for an alternate method using less equipment.)

A computer having a digital to analog converter.

An analog voltmeter.

A digital voltmeter (optional).

Instructions:

A computer program is used to generate random numbers, and these numbers are fed through a digital to analog converter to an analog voltmeter. Students read the voltmeter and type their values into the computer using the appropriate number of digits of accuracy. After several points have been inputted, the computer prints a table of the correct values, the student's values, the residuals and the mean square error. Plots may be made of read versus correct values or residuals versus correct values. Plots may also be made which compare the frequency of occurrence of least significant digits in actual and read values.

Discussion:

What are the accuracy and precision of individual students? Are some students more accurate and precise than others? How can we go about separating the precision and accuracy of the meter from that of the persons operating it? What are the accuracy and precision of the meter? Are some last digits more preferred by some students than others?

Alternate methods using less equipment:

This experiment can be done without a computer by having a variable voltage source, a digital voltmeter, and an analog voltmeter. One student sets the voltage to various RANDOM values and records the

values set while the other reads the analog meter and records those values. The student setting values should not look at the digital meter until after setting the voltage so as to avoid preferred least significant digits.

Even less equipment is required if photocopies are made of an instrument indicating various values and students are asked to read these.

Laboratory Exercise #2: Accuracy of Measurement

Objectives:

To learn about experimental errors and accuracy and sources of errors in data.

Equipment:

Masking tape, a tape measure, a 6" ruler for each pair of students.

Preparation:

Before class meets place two strips of tape on the floor. Make each strip about 60" long. Place them at an angle to floor tiles. Place the tapes so that they are as far from parallel as possible but still appear parallel. For tapes around 100" apart 2 or 3" of difference is good. Use the tape measure to measure and record the distance between the tapes and the widths of the tapes. (Warning: the janitors have been known to remove these at night.)

Instructions:

1. Have students pair up and give one 6" ruler to each pair.
2. Instructions to the students are oral and deliberately sparse. A typical statement would be:

"When you go into the hall, you will find two strips of masking tape on the floor. You are to work in pairs and use your rulers to measure the distances between the tapes. One partner will measure and the other will record work quickly, but carefully. The success of this lab depends on each group obtaining an independent measurement of the distance. Do not share your results with other groups. Do not fudge your result to try to agree with others. You should work carefully, but you will not be graded on the accuracy of your answer. Your rulers are marked in $1/32$ " (between 0 and 1). Record your answer to the nearest $1/32$ ". When you get out in the hall you may have some questions

about what to do. Deal with them as best you can. One objective of this lab is for you to have to think about some things without explicit instructions. The tapes are long enough so that two groups can measure at the same time--one on the right end and one on the left. When you are done measuring return to your seats and we will discuss the results. Are there any questions now before we go out? I will not be answering questions once we go into the hall."

Discussion:

1. How accurate do you think your result is? How many of you think you measured to within 1/16 inch?, 1/8?, 1/4?, etc., 10 inches?, 20 inches?, worse?
2. What questions did you have to deal with in making your measurements?
3. What are some factors that limit accuracy? Which do you think are most important? How could you minimize each of these?
4. Tabulate the results in categories of 1/2 inch or so. Are any values obviously wrong? How do you know this? Now how accurate do you think that the class' measurements are?
5. Some points need to be raised by the instructor if not mentioned by the class:
 - a. Did you measure inside, outside or same edges? Modify table to a common standard by adding or subtracting the width of tapes as needed.
 - b. Are both ends of the tapes the same distance apart? How could we find out? Can we determine this from the data we have without going into the hall? How? Divide the measurement table into left and right end measurements. Is there a difference between the two groups? In more sophisticated classes this could lead to discussion of statistical tests and significance.
6. How could we measure more accurately? (Some answers include: a foot ruler, a yard stick, a tape measure, a laser ranging device, an interferometer.) Are any of these "too accurate for the job"?
7. Reveal the "true" answer according to the tape measure and discuss how the class values compare with this.
8. Discuss how accuracy of data affects meteorology and some limitations on meteorological data.

Further comments and suggestions:

This laboratory can be made more relevant to meteorology by making measurements with some meteorological instrument rather than with rulers; however, the rulers illustrate many of the basic properties of measurement errors and may make the students realize that they must not take for granted even a familiar process like measuring a distance between two points.

2.2 DYNAMIC CHARACTERISTICS

2.2.1 PHYSICAL SYSTEMS AND MATHEMATICAL MODELS

We use differential equations to model the dynamic performance of systems realizing that the models can never be exact. If the dynamic behavior of physical systems can be described by linear differential equations with constant coefficients, the analysis is relatively easy because the equation solutions are well known. Such equations are always approximations to the actual performance of physical systems which are often nonlinear, time variant, and have distributed parameters. The justification for the use of simple, readily solved models must be the quality of the fit of the solution to the actual system output and the usefulness of the resulting analysis.

In this context, the word system refers to the physical device or sensor and equation refers to the corresponding mathematical model. There exists a dual set of terms corresponding to consideration of the physical system or of the mathematical model.

The general dynamic performance model is the linear ordinary differential equation:

$$a_n \frac{d^n x}{dt^n} + a_{n-1} \frac{d^{n-1} x}{dt^{n-1}} + \dots + a_1 \frac{dx}{dt} + x = x_I(t)$$

where t = time, the only independent variable,
 x = the dependent variable,
 a_n = equation coefficients or system parameters,
 $x_I(t)$ = forcing or driving or excitation function.

This equation is ordinary because there is only one independent variable. It is linear because the dependent variable and its derivatives occur to the first degree only. This excludes powers, products, and functions such as $\sin x$. If the system parameters (equation coefficients) are constant, the system is time-invariant.

Differential equations describe the behavior of physical systems in which a redistribution of energy is taking place. In a mechanical

system, a mass in motion stores kinetic energy and may store potential energy by virtue of its position in a force field. When a mechanical system does not store potential energy but does dissipate energy, the differential equation is first order in velocity, e.g.,

$$m \frac{dv}{dt} + Dv = F$$

where v = velocity,
 dv/dt = acceleration,
 m = mass,
 D = dissipation factor, and
 F = external force.

The above equation applies for a cup anemometer because the anemometer can store kinetic energy in the moment of inertia of the cup wheel, but because the cup wheel has no preferred position with respect to the wind vector, it cannot store potential energy. It dissipates kinetic energy into the wind stream.

If potential energy storage is provided in a system, then the differential equation includes a force term dependent upon displacement,

$$m \frac{d^2x}{dt^2} + D \frac{dx}{dt} + kx = F$$

where x = displacement,
 dx/dt = velocity,
 d^2x/dt^2 = acceleration,
 k = potential energy parameter.

A wind vane stores kinetic energy in the vane motion and stores potential energy when vane motion has done work against the driving wind direction. There is potential energy in the displacement of the vane with respect to the wind vector. Therefore the wind vane is modeled with a second order differential equation.

The order of the differential equation is always equal to the number of energy storage reservoirs. In a mechanical system, these reservoirs comprise the kinetic energy storage elements plus the potential energy storage elements (see Rogers and Connolly, 1960, p. 64). Capacitors and inductors are energy storage elements in electrical systems. In thermal systems, energy storage is in thermal masses.

The transient response or complementary function in mathematical terms is obtained when the forcing function is zero and the system is released from some set of initial conditions at time $t = 0$. The distribution of energy in the system storage elements at the time of

release must tend towards zero due to the always present energy dissipation factors. In system terms, the output for a given initial energy distribution and driving input is the transient solution plus the steady-state solution. In mathematical terms, the equation solution for a given set of initial conditions and a forcing function is the complementary function plus the particular function.

For a linear system, the response to a sum of inputs is simply the sum of the responses to these inputs applied separately. This is the superposition principle and can be taken as the defining property of linear systems. This is an extremely useful property because it allows analysis of the response to complex signals in the frequency domain by superposition of response to individual frequencies. This is the justification for using linear models even when the fit is far from ideal.

A physical system is said to be in a static state when the distribution of energy within the system is constant. When there is an exchange of energy within the system, the system is in a dynamic state and its performance is described by a differential equation containing derivatives with respect to time. To determine a static characteristic such as threshold, measurements of the output must be made for many different values of the input. Each measurement is made while the system is static. During the transition from one static state to another, the system is dynamic. We wait until the dynamic energy exchange has ceased before making the static measurement.

When forces are applied at discrete points and are transmitted by discrete components within the system, the system can be defined by lumped parameters. But when it is necessary to describe the variation across space coordinates of a physical component, the system must be described with distributed parameters and is modeled by a partial differential equation.

Dynamic performance analysis is concerned with modeling the performance of dynamic, lumped parameter systems with ordinary differential equations where time is the independent variable.

2.2.2 FIRST-ORDER SYSTEMS

Describing Equation

The behavior of the ideal first-order system is described by a linear first-order nonhomogeneous ordinary differential equation with one (constant) coefficient,

$$\tau \frac{dx}{dt} + x = x_I,$$

where

$x(t)$ = output (response) function, the dependent variable,

$x_I(t)$ = input (forcing) function,

t = time, the independent variable,

and

τ = time constant, the system dynamic performance parameter.

To obtain the response $x(t)$, an initial condition $x(0)$ and a particular input function $x_I(t)$ must be specified. The combination of the describing differential equation and the initial condition form an initial value problem.

Methods of Solution

The solution will, in general, consist of two independent parts:

$$x(t) = x_p(t) + x_c(t),$$

where $x_p(t)$ = particular solution (sometimes called the "steady-state response" or the "final state"); determined by the functional form of $x_I(t)$,

$x_c(t)$ = complementary solution (usually called the "transient response"); it contains one constant to be determined from the initial $x(0)$.

Transform Methods. These include "operator notation," Fourier transform, and Laplace transform methods. Transform methods are particularly useful when investigating the frequency response of a system as they allow a mapping from the time domain to the frequency domain. Transform methods facilitate the representation of a system in terms of a block diagram, a powerful graphic tool for describing system functioning.

Teaching Note. Most undergraduate meteorology students will not be familiar with transform methods for solving ordinary differential equations. The integrating factor approach (outlined below) for solving first-order ordinary differential equations should be more familiar to them and so is perhaps easier to use in a first course.

Integrating Factor. The integrating factor approach, a straightforward mathematical technique used in solving a first-order ordinary differential equation, allows one to state the general solution to the equation in a manner that clearly shows the roles of the forcing function and the initial condition in determining the particular and complementary solutions, respectively. The goal of this technique is to find a function of $\mu(t)$. The product of this function, with the lefthand side of the describing differential equation, results in an exact differential. Then for the lefthand side of the describing differential equation,

$$\mu(t)\left(\frac{dx}{dt} + \frac{x}{\tau}\right) = \frac{d}{dt}[\mu(t)x(t)].$$

While in general such a function is not easily found, here, with τ a constant, $\mu(t)$ is readily determined to be

$$\mu(t) = e^{t/\tau}.$$

Using this, the following explicit formula for the solution to the describing ordinary differential equation is obtained by

$$x(t) = e^{-t/\tau} \left[\frac{1}{\tau} \int_0^t e^{r/\tau} x_I(r) dr + C \right].$$

(A) (B)

Here r = dummy variable of integration,

and C = constant of integration.

The (A)-term is the particular solution, while the (B)-term is the complementary solution.

Time Response

Here the focus is on the evolution of the system output as a function of time, that is, on the initial or transient phase. As inputs, it is conventional to consider both a step function (a common test signal used in many laboratory procedures) and a ramp function (which is a more realistic description of the type of signals encountered in many meteorological processes). In each case, the initial state (condition) of the system will be taken to be steady in equilibrium with the input. That is,

$$x(t) = x_I(t) = x_0, \quad t < 0,$$

where x_0 = initial value (sometimes called the "initial offset"), a constant.

Step Input. In this case, the input is initially at x_0 , but at time $t = 0$, it changes instantaneously by an amount $(x_\infty - x_0)$ to a new value x_∞ .

Teaching Note. The quantity x_∞ will be used throughout in the definition of input functions. Its physical meaning will vary depending on the functional form of the input signal.

In functional form

$$x_I(t) = \begin{cases} x_0, & t < 0, \\ x_\infty, & t > 0. \end{cases}$$

Note that this input function is not explicitly defined at $t = 0$.

The response of the system for $t > 0$ is

$$x(t) = x_0 + (x_\infty - x_0)(1 - e^{-t/\tau}).$$

(A) (B)

Here, the (A)-term yields the new state of equilibrium approached as $t \rightarrow \infty$. The (B)-term is the transient, which vanishes as $t \rightarrow \infty$. Note that the time, t , occurs in a dimensionless ratio with the time constant, τ , and further, that the response term can be written in terms of differences between the current and final states and the initial state. These points suggest that it might be convenient to cast this solution into a dimensionless, normalized form for discussion and plotting purposes:

$$x'(t') = 1 - e^{-t'},$$

where

$$x'(t') = \frac{x(t) - x_0}{x_\infty - x_0},$$

$$t' = \frac{t}{\tau}.$$

In this standard form, the offset represented by x_0 has been removed, and the response has been normalized with respect to the amplitude of the input signal. The time has been scaled with the characteristic time appropriate to this system, time constant τ . An advantage to this

form is that the output function $x'(t')$ is independent of the particular values of x_0 , x_∞ , and τ .

Teaching Note. Using this same scheme, the nondimensionalized, normalized form of the input function becomes

$$x_I'(t') = \begin{cases} 0, & t' < 0, \\ 1, & t' > 0. \end{cases}$$

Thus for a step change, the first-order system adjusts to its new state as shown in Fig. 2-6; some numerical values are given in Table 2-1. After about five units of dimensionless time t' , the transition to the new equilibrium state is essentially completed. The dimensionless dynamic error $\varepsilon_D(t') = x'(t') - x_I'(t')$.

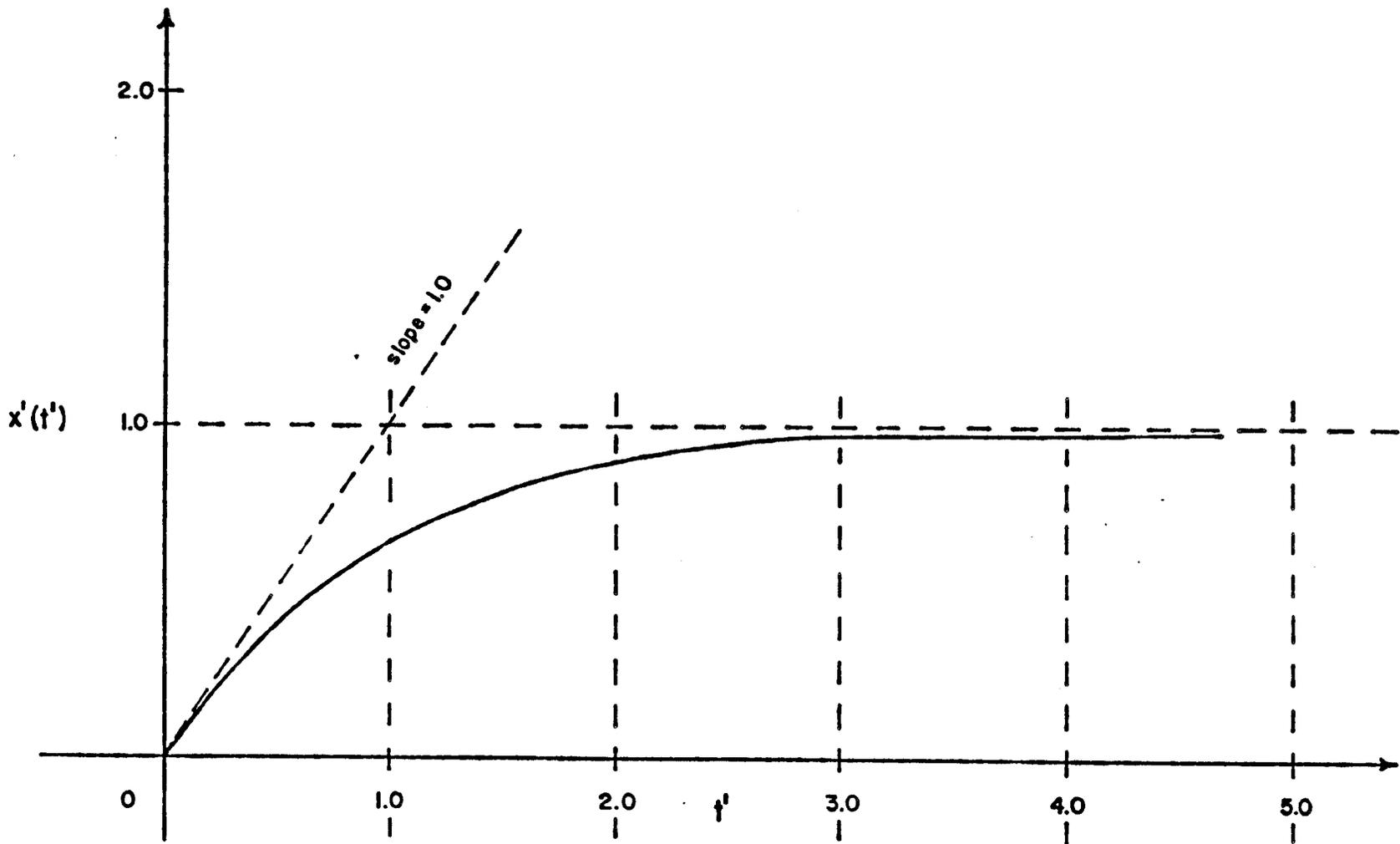
Table 2-1. Step Response

t'	$x_I'(t')$	$x'(t')$	$\varepsilon_D(t')$
0+	1.000	0.000	-1.000
1	1.000	0.632	-0.368
2	1.000	0.865	-0.135
3	1.000	0.950	-0.050
4	1.000	0.982	-0.018
5	1.000	0.993	-0.007

Particular points to note in Fig. 2-6 include:

1. The new final state is one of constant value equal to the input.
2. There is a discontinuity in the slope of the response curve at $t' = 0$. (At $t' = 0^-$, the slope is zero, while at $t' = 0^+$, the slope is unity.) This discontinuous change in slope is a direct consequence of the instantaneous change in the input at $t' = 0$ and cannot occur in the response of a real system. Further, the instantaneous change at $t' = 0$ required by an exact "step" function cannot be generated in the physical world. These two features are results of idealizations made in the system model and in the input function, respectively.

Fig. 2-6. Step function response of a first-order system.



3. For any point along the response curve, the difference between the value of the output at that point and the difference of the final value will decrease by $\approx 63\%$ during the next unit of dimensionless time t' .

4. The slope of the response curve, the time-rate-of-change of $x'(t')$, is initially unity and then decreases exponentially with time. Thus the initial magnitude of the time-rate-of-change of the dimensional, nonnormalized response depends directly on the magnitude of the change, $(x_\infty - x_0)$, and inversely on the system parameter τ , becoming slower as the ratio

$$\frac{x_\infty - x_0}{\tau}$$

decreases and faster as the ratio increases.

Ramp Input. Again the input is taken to have an initial value of x_0 . Beginning at $t = 0$, it changes linearly from this initial value at a rate

$$\frac{(x_\infty - x_0)}{\tau}.$$

In functional form

$$x_I(t) = \begin{cases} x_0, & t < 0, \\ x_0 + (x_\infty - x_0)\left(\frac{t}{\tau}\right), & t > 0. \end{cases}$$

The corresponding response is

$$x(t) = x_0 + [(x_\infty - x_0) \frac{(t - \tau)}{\tau}] + (x_\infty - x_0)e^{-t/\tau}.$$

(A) (B)

Here (A) represents the final, linearly time-dependent state of the system, approached at $t \rightarrow \infty$. The (B)-term is again the transient, which vanishes as $t \rightarrow \infty$. In dimensionless, normalized form, these expressions become

$$x_I'(t') = \begin{cases} 0, & t' < 0, \\ t', & t' > 0, \end{cases}$$

and

$$x'(t') = (t' - 1) + e^{-t'}.$$

This last is plotted in Fig. 2-7. As shown in Table 2-2, again after about five units of dimensionless time, the transient has become negligibly small, with the system response closely following the asymptote indicated in Fig. 2-7.

Table 2-2. Ramp Response

t'	$x'(t')$ I	$x'(t')$	$\epsilon(t')$ D
0+	0.000	0.000	0.000
1	1.000	0.368	-0.632
2	2.000	1.135	-0.865
3	3.000	2.050	-0.950
4	4.000	3.018	-0.982
5	5.000	4.007	-0.993

Particular points to note in Fig. 2-7 include:

1. The new state (for $t' \gg 1$) is one with uniform rate of change of output with time. The time-rate-of-change of the output is the same as that of the input.

2. At any instant, the output magnitude (for $t' \gg 1$) is less than the input value by a unit amount. This corresponds to a dimensional, nonnormalized difference in magnitude between input and output equal to the product of the time-rate-of-change of the input and the time constant, that is, to a difference of $(x_\infty - x_0)$.

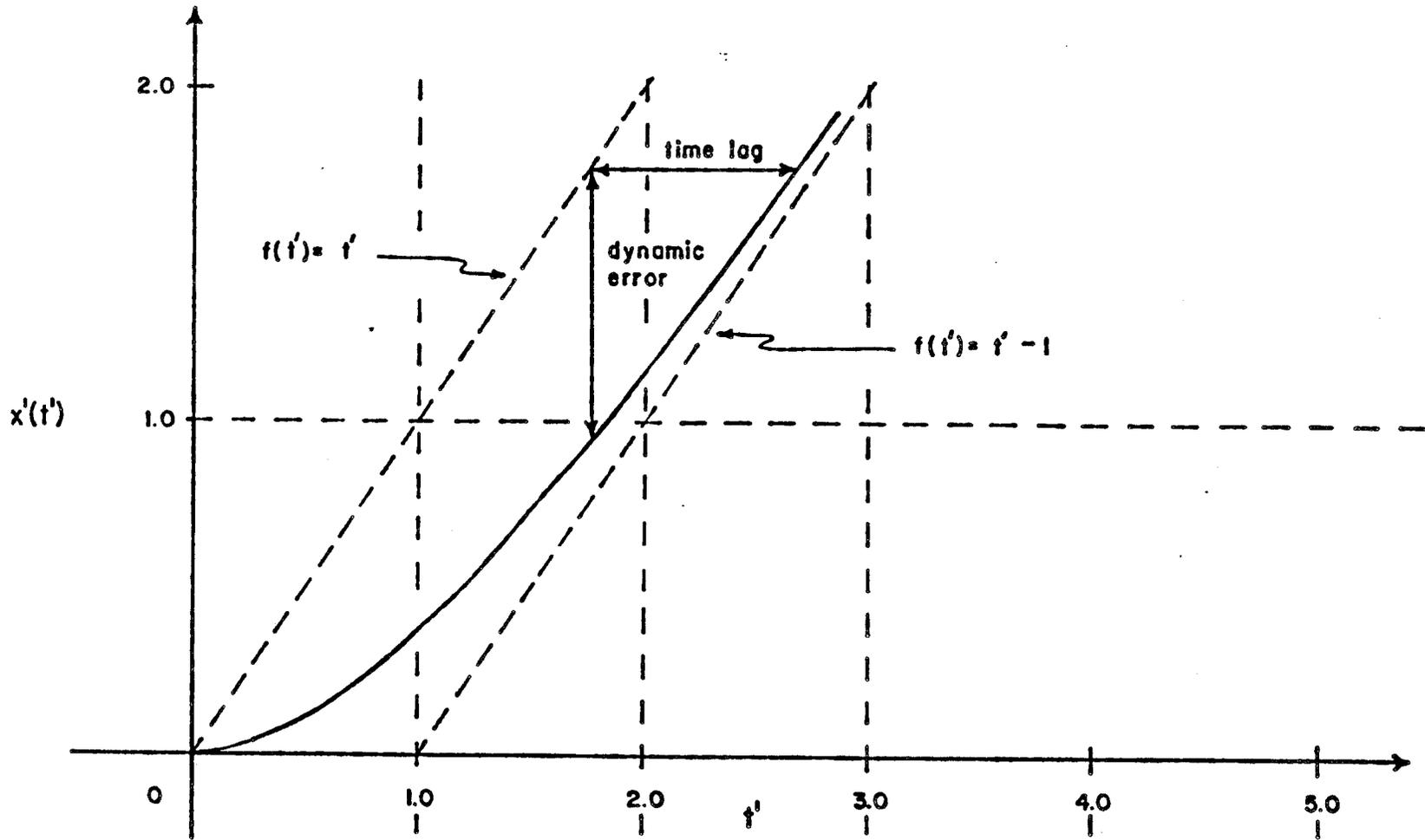
3. The final state (for $t' \gg 1$) is one in which the output lags the input by one unit of dimensionless time. In a dimensional sense, this means that the output takes on a particular value at τ units of time after the input had that same value.

4. The initial slope of the response curve is zero. This reflects the continuity of the input ramp function at $t' = 0$.

Dynamic Characteristics, Part I

The time response of a first-order system can be used to illustrate several important dynamic characteristics. With proper interpretation, all the concepts presented here can be applied to more complicated systems.

Fig. 2-7. Ramp response of a first-order system.



Settling Time. The settling time t_s is the interval of time required (after the application of a change in the input) for the output to reach and then to stay within a stated plus-and-minus tolerance band around the final state. It is thus a measure of the duration of the transient phase of the response. A small settling time is indicative of a fast-responding instrument. Note that there is a certain degree of arbitrariness in this definition because of the requirement that the user specify the width of the plus-and-minus tolerance band.

As an example, for a first-order system, for a +5% tolerance band, $t_s' = 3$; for a +1% band, $t_s' \approx 4.5$. (Note that the non-dimensionalization of time with the time constant has been carried over in this statement.) These are most easily seen in the case of the step input. However, these two values are equally valid for the ramp input where the final state is one of linear change in output with time.

Lag (sometimes called "retardation" or "time delay"). The term "lag" is sometimes used synonymously with "settling time" to describe the duration of the transient phase of the response. However, this is poor practice as confusion can result. It is better to use settling time in discussing the transient phase and to reserve the term lag for use in characterizing the final state response ($x'(t')$ when $t' \gg 1$). In this sense, the lag time t_l in the final response is the time difference between when a specific value of output is obtained and when that same value was previously attained by the input.

In the case of the ramp input function, the lag time is numerically equal to the time constant, that is,

$$t_l' = 1.$$

After the transient portion of the response vanishes, the system output lags the input in time by this constant amount. Note that in this case, two system characteristics (settling time and lag time) must be specified to describe the response. This is directly connected to the time-varying nature of the input function.

On the other hand, in the manner in which the term lag has been defined here, there is no final state lag in the case of the step input. For $t' \gg 1$, the output effectively equals the input. Only a single quantity, the settling time, is needed to characterize the response to a step input.

Dynamic Error. The dynamic error is the difference between the value output function at each instant in time and the value of the input function (a quantity normally changing with time). It is best expressed in a dimensionless, normalized form, such as

$$\epsilon_D(t') = x'(t') - x_I'(t').$$

For the step input,

$$\varepsilon_D(t') = -e^{-t'}$$

In this case, the dynamic error is a transient quantity, reflecting the exponential approach of the output to the constant input value with time. Note that

$$\lim_{t' \rightarrow 0} \varepsilon_D(t') = -1,$$

and

$$\lim_{t' \rightarrow \infty} \varepsilon_D(t') = 0.$$

Values for the dynamic error for a step input, as function of dimensionless time, are given in Table 2-1.

For the ramp input,

$$\varepsilon_D(t') = -1 + e^{-t'}.$$

(A) (B)

This consists of both a steady-state error, the A-term, and a transient error, the B-term. The dynamic error is initially zero but ultimately takes on a unit value reflecting the lag in the system response. Note that

$$\lim_{t' \rightarrow 0} \varepsilon_D(t') = 0,$$

and

$$\lim_{t' \rightarrow \infty} \varepsilon_D(t') = -1.$$

Values for the dynamic error in this case are given in Table 2-2.

In the dimensional, nonnormalized sense, the smaller the time constant, the quicker the transient error disappears (true for both cases). On the other hand, the steady-state error is directly proportional to the product of the time constant with the time-rate-of-change of the input, that is, to $(x_\infty - x_0)$. Hence, the quicker the input changes, measured relative to the speed at which the system can respond, the larger the steady-state error.

Frequency Response

Here the focus is on the steady-state output response when the input is a sinusoidal oscillation. The reason for special interest in the response of the system to a sinusoidal input is that it provides the basis for assessing the system's response to much more complicated input signals. Through the use of Fourier Series (in the case of

periodic inputs) or the Fourier Integral (in the case of aperiodic inputs), an input signal can be described in terms of its amplitude and phase spectra. Further, because the describing differential equation is linear, the Principle of Superposition can be applied. Consequently, we can obtain the response of the system to a complicated input (one having many spectral components, each described by an amplitude and a phase angle) by decomposing the input signal into its spectral components, finding the response of the system to each of these components individually, one at a time, and then finally summing to synthesize the total output signal. In passing through the system, each of the individual spectral components of the input signal will, in general, be modified in amplitude and shifted in phase. For this linear system, each spectral component of the input signal produces only a single spectral component, at the same angular frequency, in the total output signal.

A little reflection on this process leads to the conclusion that an ideal measuring system would be one which produces an output response signal in which the spectral components have the same relative amplitude and relative phase angle spectra as the input signal. That is, in the course of processing the signal, the ideal measuring system would at most:

-- Change the amplitude of each spectral component by the same, fixed amount. The amplification or attenuation of the signal components should be independent of frequency. The amplitude response of such a system is said to be "flat."

-- Change the phase angle of each spectral component by an amount directly proportional to the input angular frequency. The time delay experienced by each spectral component, in passing through the system, would then be independent of the input angular frequency. The phase response of such a system is said to be "linear."

Sinusoidal Input. With these two points in mind, we turn to a consideration of the response of the first-order measuring system to a sinusoidal input. For the purposes of this analysis, the initial state of the system is again taken to be one of constant equilibrium, with

$$x(t) = x_I(t) = x_0, \quad t < 0.$$

Set

$$x_I(t) = \begin{cases} x_0, & t < 0, \\ x_0 + (x_\infty - x_0)\sin(\omega_I t), & t > 0. \end{cases}$$

Here $(x_\infty - x_0) =$ input amplitude, a constant,

and $\omega_I =$ input angular frequency.

It is also convenient to rewrite the controlling first-order differential equation as

$$\frac{dx}{dt} + \omega_b x = \omega_b x_I,$$

where $\omega_b = \frac{1}{\tau}$,

= breakpoint angular frequency.

The response of the system to this input is

$$x(t) = x_0 + (x_\infty - x_0) \frac{\omega_b}{(\omega_b^2 + \omega_I^2)^{1/2}} \left\{ \sin[\omega_I t - \tan^{-1}(\frac{\omega_I}{\omega_b})] + e^{-\omega_b t} \sin(\omega_I t) \right\}.$$

As this form is cumbersome, it is again helpful to non-dimensionalize and normalize the result. Using the same definitions introduced previously in Section 2.2.2 under Time Response, the input function becomes

$$x'(t') = \begin{cases} 0, & t' < 0, \\ 1 + \sin(\alpha t'), & t' > 0, \end{cases}$$

while the response function becomes

$$x'(t') = A(\alpha) \{ \sin[\alpha t' - \phi(\alpha)] +$$

(A)

$$e^{-\alpha t'} \sin[\phi(\alpha)] \}.$$

(B)

Here
$$\alpha = \frac{\omega_I}{\omega_b},$$

= dimensionless frequency (sometimes called the frequency number),

$$A(\alpha) = \frac{1}{(1 + \alpha^2)^{1/2}}$$

= normalized output amplitude function,

and
$$\phi(\alpha) = \tan^{-1}(\alpha),$$

= phase angle function.

The above expression again includes both a steady-state portion (A) and a transient portion (B). That the output angular frequency is the same as that of the input is a direct consequence of the linearity of the describing differential equation. The input and output functions are illustrated in Fig. 2-8. After a period of time equal to a few time constants, the transient portion becomes negligibly small, leaving the sinusoidally varying final state. This is the portion of the response that is of most interest. It is characterized by the output amplitude function $A(\alpha)$ and the phase shift angle $\phi(\alpha)$, which is sometimes called the phase lag or the phase delay. Plots of these two functions are shown in Figs. 2-9 and 2-10.

The phase angle indicates by what angle the response attains its maximum, relative to the occurrence of the maximum in the input signal. The inclusion of a minus sign with the phase angle in the argument of the oscillating sine term in the response function means that a positive value of $\phi(\alpha)$ results in the output lagging behind the input signal.

Teaching Note. It might be helpful to the student to point out that the argument of a trigonometric function is always an angle and thus dimensionless. An alternate form for the the argument of the sine term in the above response function is

$$[\alpha(t' - t'_{\text{shift}})],$$

where t'_{shift} is defined below. This form might help the student in grasping the idea of the delay represented by the phase angle $\phi(\alpha)$.

Important points to note in these last two figures include:

Fig. 2-8. Time series of sinusoidal input and first-order response.

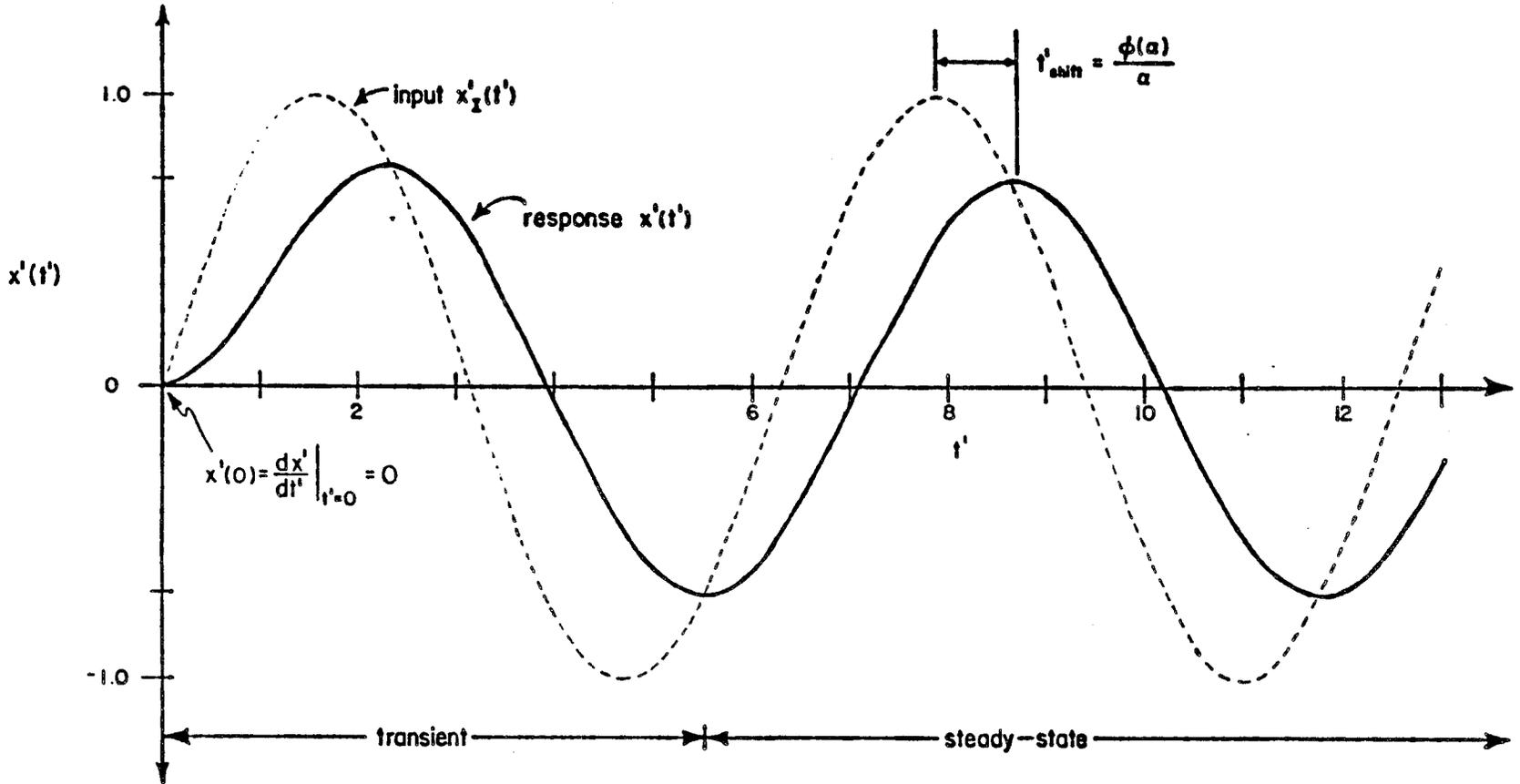


Fig. 2-9. Amplitude function of first-order response to a sinusoidal input.

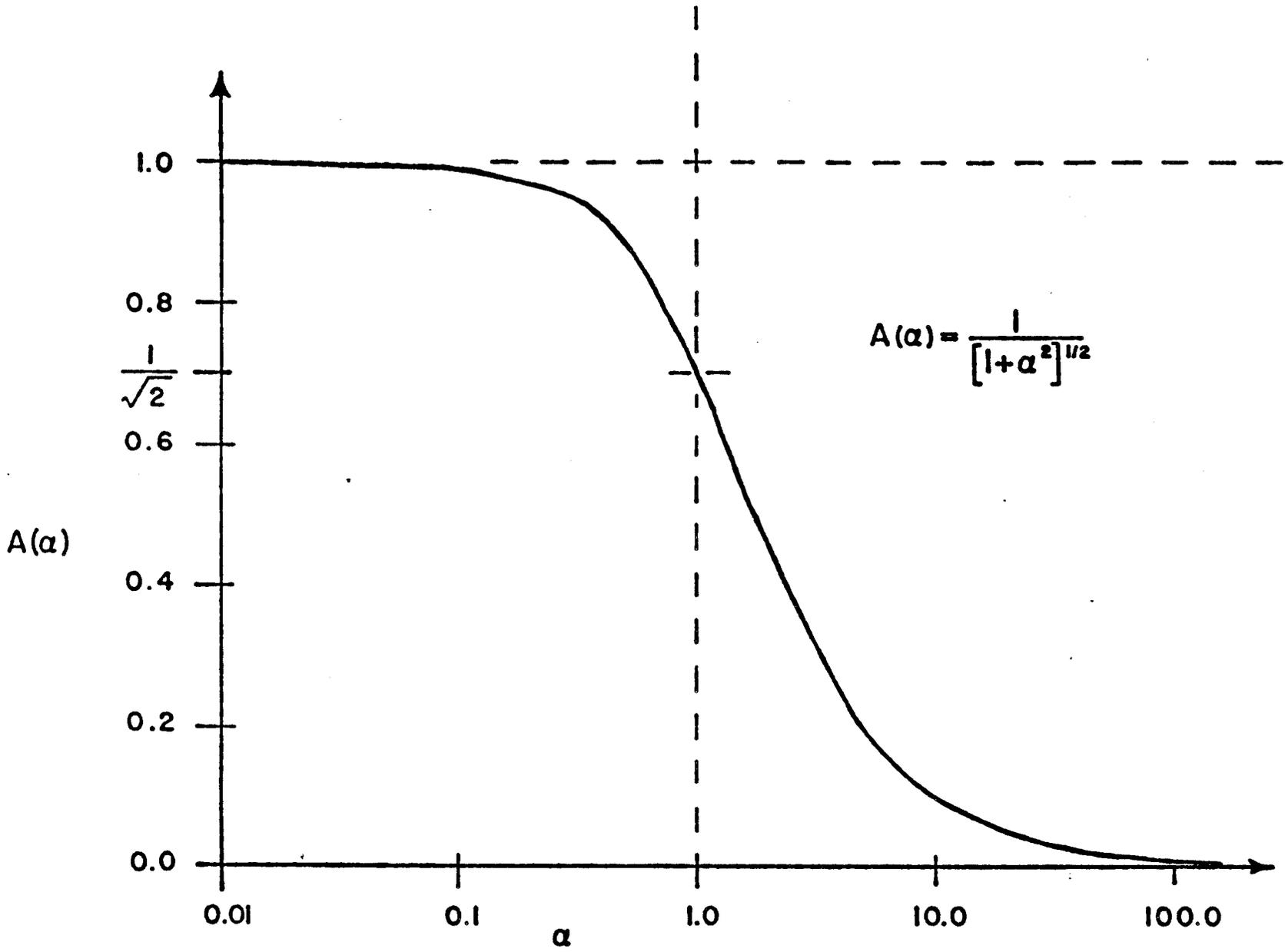
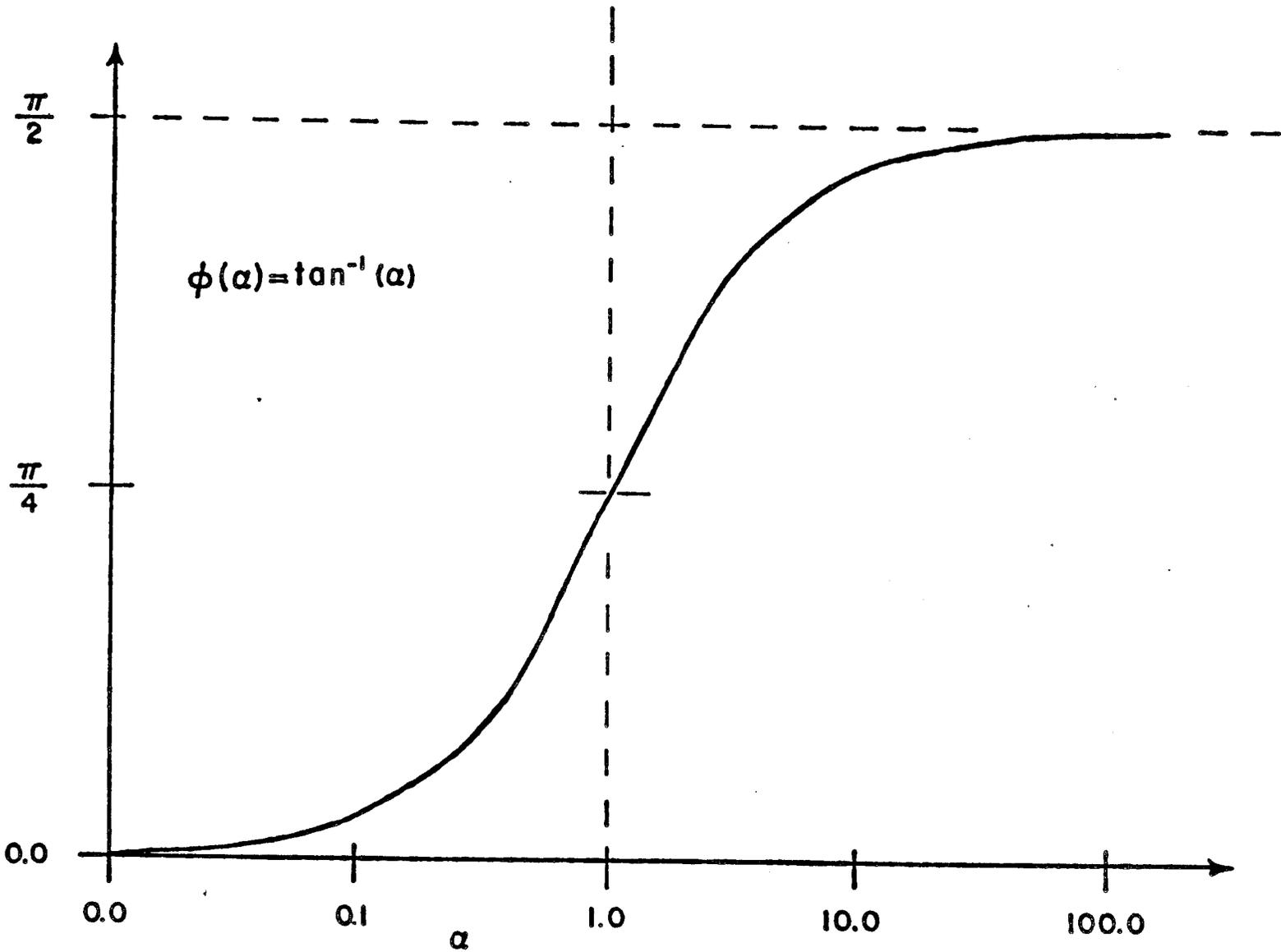


Fig. 2-10. Phase function of first-order response to a sinusoidal input.



1. In the low frequency limit,

$$\lim_{\alpha \rightarrow 0} A(\alpha) = 1, \text{ and } \lim_{\alpha \rightarrow 0} \phi(\alpha) = 0,$$

that is, the amplitude of the change in $x'(t')$ is the same as for a static displacement. The system successfully follows very slowly changing input signals.

2. In the high frequency limit,

$$\lim_{\alpha \rightarrow \infty} A(\alpha) = 0, \text{ and } \lim_{\alpha \rightarrow \infty} \phi(\alpha) = \frac{\pi}{2},$$

that is, the amplitude vanishes as the system is too sluggish to follow the rapidly varying input signals.

3. At $\alpha = 1$, the amplitude ratio has value

$$A(1) = 2^{-1/2}, \text{ while } \phi(1) = \frac{\pi}{4}.$$

Recall that the power contained in a particular spectral component is directly proportional to the square of the amplitude of that component. Hence, at the breakpoint frequency, the output power of the corresponding spectral component will be reduced by one-half (the breakpoint frequency is sometimes called the half-power point). Spectral components at frequencies greater than the breakpoint will undergo even stronger reduction in power content. For this reason, the breakpoint or half-power point frequency is often cited in describing the frequency response of a system.

4. In terms of reproducing the proper signal shape at the output, a first-order measuring system has relatively poor amplitude and phase shift characteristics. It is essentially a poor quality low-pass filter. That is, it passes only very low frequency signals in undistorted form. At input frequencies of $\alpha = 0(1)$ and larger, strong attenuation and severe phase distortion of the output occurs.

An Illustrative Example. The physical basis for the behavior of a first-order system in response to a sinusoidal input, predicted by the foregoing mathematical analysis, can perhaps be best visualized by considering a simple spring-dashpot mechanical system. A spring is a potential energy storage device. When charged with energy through extension or compression from an equilibrium length, a spring produces a force. This "restoring" force (F_{spring}) is directed toward returning the spring to its equilibrium length and has a magnitude linearly proportional to the degree of extension/compression. That is, taking one end of the spring to be fixed,

$$F_{\text{spring}} = -Kx,$$

where x is the displacement of the free end from equilibrium. A dashpot is an energy absorbing device, usually dissipating energy through work done against friction. It provides a resistive force (F_{dashpot}) with magnitude linearly proportional to the time-rate-of-change of displacement and directed so as to oppose the motion. That is,

$$F_{\text{dashpot}} = -R \frac{dx}{dt} .$$

Note that the restoring force provided by the spring always lags the displacement by π (180°), while the resistive/dissipative force provided by the dashpot always lags the displacement by $\pi/2$ (90°).

The dynamic force balance for this system requires that at each instant of time

$$F_{\text{spring}} + F_{\text{dashpot}} + F_I = 0.$$

Consider the situation with $\alpha \ll 1$. Here the applied force $F_I(t)$ is almost in phase with the displacement $x(t)$. $F_I(t)$ will thus lead the restoring force provided by the spring by π and the resistive force of the dashpot by $\pi/2$. However, the time-rate-of-change of the signal is small, so the magnitude of the resistive force is small. The dynamic force balance is maintained by the restoring force opposing the applied force; the resistive force makes only a small contribution. To obtain the necessary magnitude of restoring force, a significant displacement can occur.

For $\alpha \gg 1$, the applied force $F_I(t)$ leads the displacement by $\pi/2$. $F_I(t)$ will thus lead the spring restoring force by $3\pi/2$ (it would be equally valid to say that the restoring force leads the applied force by $\pi/2$) and the resistive force by π . In this case, the magnitude of the resistive force is large because the time-rate-of-change of the input signal is large; displacements are small so that the magnitude of the restoring force is small. Thus the dynamic force balance is maintained by the resistive force opposing the applied force.

Dynamic Characteristics, Part II

Amplitude Distortion. For an ideal measuring system, we should have

$$A(\alpha) = K, \text{ a constant, for all } \alpha.$$

That is, the amplitude of an ideal system is "flat" over the full range of frequencies, $0 < \alpha < \infty$. In practice, even the best real measuring system has an amplitude response function that is only approximately flat over a limited range of input frequencies.

The amplitude function for a first-order system is such that

$$0 < A(\alpha) < 1, \text{ for all } 0 < \alpha < \infty,$$

that is, the amplitude of the output is always reduced (attenuated) to a greater or lesser extent with respect to that of the input. Most importantly, this is a frequency-dependent effect.

Examination of Fig. 2-9 shows that the amplitude response of the first-order system approximates the ideal flat response only for very small input frequencies. Consequently, significant distortion should be expected in the composite output signal due to the loss of amplitude in the high frequency portion of the input signal spectrum. The output will have a "rounded-off" or smoothed appearance.

Formally, amplitude distortion is defined as the effect of a frequency-dependent output amplitude function acting to change the relative amplitudes of the spectral components of the signal as it is processed through the system.

Phase Distortion (Phase Shift). In general, a sinusoidal signal will be delayed while being processed by a measuring system. That is, the output will lag behind the input. It is conventional to express this delay in terms of the phase shift angle $\phi(\alpha)$. The corresponding shift in (dimensionless) time of the output with respect to the input is given by

$$t'_{\text{shift}}(\alpha) = \frac{\phi(\alpha)}{\alpha},$$

where a possible frequency dependence of the shift is indicated.

Again, for an ideal measuring system, we should have

$$\phi(\alpha) = \alpha t'_{\text{shift}}, \text{ where } t'_{\text{shift}} = \text{constant.}$$

That is, the ideal measuring system has a "linear phase response" over the full range of frequencies, $0 < \phi < \infty$. This guarantees that each spectral component of the signal experiences the same temporal delay in passing through the system. In practice, real systems will at best have approximate linear phase response over only a limited range of input frequencies.

By referring to Fig. 2-10, the first-order system is seen to approximate the desired linear phase shift condition only for very small input frequencies. Significant distortion in the composite output signal should be expected as the high frequency components will be shifted in time, relative to one another.

Phase distortion is defined as the effect of a frequency dependent phase shift function acting to delay, relative to one another, the

various spectral components of the signal as it is processed by the system.

Dynamic Error. Dynamic error results from a combination of amplitude and phase distortion of the signal as it is processed through the system. Only in the limit of very small input frequency will the form of the output signal from a first-order measuring system be the same as that of the input.

Using these ideas, it is possible to see why the response of a first-order system in a step input is "rounded-off" into the observed exponential transition from one state to another. A Fourier Integral representation of a step change shows that most of the information about the sharp change at $t' = 0$ is contained in the amplitudes of the high frequency spectral components, while the low frequency components contain the information about the new final value. Because both the amplitude and the phase distortion of the spectral components of the step signal occur as they are processed by a first-order system, much of the high frequency information about the sharp leading edge is lost. Consequently, the output signal contains mostly low frequencies and the leading edge of the output signal is rounded-off.

2.2.3 SECOND-ORDER SYSTEMS

Describing Equation

The behavior of the ideal second-order system is described by a linear second-order nonhomogeneous ordinary differential equation, with two (constant) coefficients,

$$\frac{d^2x}{dt^2} + 2\zeta\omega_n \frac{dx}{dt} + \omega_n^2 x = \omega_n^2 x_I$$

where $x(t)$ = output (response) function, the dependent variable,

$x_I(t)$ = input (forcing) function,

t = time, the independent variable,

ζ = damping ratio, a constant,

and ω_n = undamped natural frequency, a constant.

These last two quantities, ζ and ω_n , are the dynamic performance parameters for a second-order system. To obtain a general solution, two initial conditions, $x(0)$ and $dx/dt|_0$, and a particular input

function $x_I(t)$ must be specified. We are again concerned with solving an initial value problem.

Methods of Solution

The general solution will consist of two independent parts:

$$x(t) = x_p(t) + x_c(t),$$

where $x_p(t)$ = particular solution: determined by the input function $x_I(t)$,

$x_c(t)$ = complementary solution: the solution to the equivalent homogeneous equation, itself having two independent parts and containing two constants to be determined from the initial conditions.

Transform Methods. The remarks made in Section 2.2.2 concerning transform methods apply here also.

Teaching Note. The technique to be used to solve for the particular solution of this second-order equation is termed variation of parameters. This should be familiar to most meteorology students who have completed their first course in differential equations.

Variation of Parameters. This technique uses the complementary solution $x_c(t)$ to construct the particular solution $x_p(t)$.

Complementary Solution: The complementary solution is the solution to the describing differential equation with the right-hand side (the forcing term) set equal to zero. As the describing ordinary differential equation is second-order, this portion of the general solution will have the functional form

$$x_c(t) = Ax_{c1}(t) + Bx_{c2}(t)$$

where $x_{c1}(t), x_{c2}(t)$ = two linearly independent solutions to the homogeneous form of the describing differential equation,

and A, B = constants to be determined by the initial conditions.

The simplest way to determine the functions $x_{c1}(t)$ and $x_{c2}(t)$ is to assume a time dependence of the form e^{rt} , substitute it into

the describing differential equation, and then find the pair of roots of the resulting characteristic equation,

$$r^2 + 2\zeta\omega_n r + \omega_n^2 = 0.$$

Each root of this algebraic equation gives rise to a solution. Depending on the value of the damping ratio, one of four different possibilities for the roots of this equation will occur. Consequently, four different functional forms for $x_c(t)$ are possible.

1. $\zeta = 0$ --two imaginary, unrepeated roots (gives rise to free oscillations):

$$x_c(t) = C \sin(\omega_n t + \theta),$$

where $C =$ amplitude constant,

and $\theta =$ phase angle, a constant.

This result shows why ω_n is called the undamped natural frequency. It is the angular frequency at which the system would oscillate in the limit of zero damping. Note that in this case the system, without damping, would oscillate indefinitely.

2. $0 < \zeta < 1$ --two complex, unrepeated roots (gives rise to damped oscillations):

$$x_c(t) = C e^{-(\zeta\omega_n t)} \sin(\omega_m t + \theta),$$

where $\omega_m = \omega_n (1 - \zeta^2)^{1/2},$

$=$ modified (damped) natural frequency,

$C =$ amplitude constant,

and $\theta =$ phase angle, a constant.

This result shows why ω_m is called the modified or damped natural frequency. It is the angular frequency at which the system would oscillate with some degree of damping present, so long as $0 < \zeta < 1.0$.

3. $\zeta = 1$ --two real, repeated roots (gives rise to a critically damped solution, one with no oscillations):

$$x_c(t) = e^{-(\omega_n t)} (At + B),$$

where $A, B =$ constants.

4. $\zeta > 1$ --two real, unrepeated roots (gives rise to an overdamped solution):

$$x_c(t) = e^{-(\zeta\omega_n t)} [Ae^{+(t/\tau_m)} + Be^{-(t/\tau_m)}],$$

where

$$\tau_m = \frac{1}{\omega_m},$$

and

A, B = constants.

A point to note in each of the four expressions is that ω_n and t appear only on the right-hand side in the form of the dimensionless product $\omega_n t$. This indicates that ω_n^{-1} is the characteristic time scale for this system and suggests defining the dimensionless time

$$t'' = \omega_n t$$

for later use.

Teaching Note. In the overdamped case, it is convenient to consider two alternative forms. First, define

v = damping number,

$$= \frac{1 + (1 - \zeta^{-2})^{1/2}}{1 - (1 - \zeta^{-2})^{1/2}},$$

with

$$\zeta = \frac{v + 1}{2 v^{1/2}}.$$

By introducing the damping number, both a simpler form for the overdamped expression and a more easily managed working range of values for the characterizing parameter are obtained. That is, we will have values like $v = 5.0$ and 10.0 rather than $\zeta = 1.3416\dots$ and $1.7393\dots$, respectively. In this formulation,

$$x_c(t) = Ae^{-\omega_n v^{1/2} t} + Be^{-\omega_n v^{-1/2} t}.$$

The dynamic performance parameters are now ω_n and v .

Second, for purposes of physical interpretation, it is advantageous to use an alternative form of the describing differential equation. Set

$$\tau_1 = \frac{[\zeta + (\zeta^2 - 1)^{1/2}]}{\omega_n} = \frac{\nu^{1/2}}{\omega_n},$$

and

$$\tau_2 = \frac{[\zeta - (\zeta^2 - 1)^{1/2}]}{\omega_n} = \frac{1}{\omega_n \nu^{1/2}},$$

with

$$\omega_n^2 = \frac{1}{\tau_1 \tau_2},$$

and

$$\zeta = \frac{\tau_1 + \tau_2}{2 (\tau_1 \tau_2)^{1/2}}.$$

The describing differential equation becomes

$$\frac{d^2x}{dt^2} + \frac{(\tau_1 + \tau_2)}{\tau_1 \tau_2} \frac{dx}{dt} + \frac{1}{\tau_1 \tau_2} x = \frac{x_I(t)}{\tau_1 \tau_2}$$

where the dynamic performance parameters are now the two time constants τ_1 and τ_2 . The corresponding complementary solution for the overdamped case takes on the particularly simple form

$$x(t) = Ae^{-(t/\tau_1)} + Be^{-(t/\tau_2)}.$$

The system in this formulation shows two characteristic time scales. As $\tau_1 > \tau_2$ always and frequently $\tau_1 \gg \tau_2$, the first term on the right-hand side will generally be significant much longer than the second.

The complementary solutions for two of these cases, the ones with zero damping ratio (free oscillations) and the ones with unit damping ratio (critically damped), represent mathematical idealizations. It is not physically possible to build real systems with damping ratios of exactly zero and exactly unity. Consequently, it is sufficient for practical purposes to focus attention on the underdamped and overdamped cases, recognizing the other two as limiting situations.

Particular Solution: Applying the variation of parameters technique, the particular solution $x_p(t)$ is obtained by using the appropriate complementary solution $x_c(t)$.

$$x_p(t) = -\omega_m^2 x_{c1}(t) \int^t \frac{x_{c2}(r) x_I(r)}{W[x_{c1}(r), x_{c2}(r)]} dr$$

$$+ \omega_m^2 x_{c2}(t) \int^t \frac{x_{c1}(r) x_I(r)}{W[x_{c1}(r), x_{c2}(r)]} dr,$$

where the Wronskian

$$W[x_{c1}(t), x_{c2}(t)] = x_{c1} \frac{dx_{c2}}{dt} - \frac{dx_{c1}}{dt} x_{c2}.$$

Time Response

As in Section 2.2.2 concerning time response, the focus in this section is on the evolution of the transient portion of the system output. The discussion here will center on the underdamped and overdamped responses to both step and ramp inputs. In each case, the initial state of the system will be a steady one in equilibrium with the input. That is,

$$x(0) = x_I(0) = x_0,$$

and

$$\left. \frac{dx}{dt} \right|_0 = \left. \frac{dx_I}{dt} \right|_0 = 0.$$

The functional forms of the step and ramp inputs will be the same as those in Section 2.2.2 and will not be repeated here.

Teaching Note. Review of first-order systems and a little physical reasoning can save much mathematical effort at this point. For three of the possible cases (underdamped, critically damped, and overdamped), one can anticipate obtaining the same particular solution $x_p(t)$. This is to be expected because the final state resulting from the application of a given input signal will be the same in each case. The details of the transient adjustment phase will differ, but these are all contained in $x_p(t)$ and are set by using the initial conditions after $x_c(t)$ is obtained.

The behavior of the fourth case (undamped) is not so clear because continuing, undiminishing oscillations are expected. As it turns out, $x_p(t)$ is the same here also.

Because the functional form of $x_c(t)$ is perhaps the simplest for the case of critical damping, it is often easiest to solve this case for $x_p(t)$ and then to apply this result to the other cases.

Step Input. In dimensionless normalized form, the response to a step input is:

1. For $\zeta = 0$, no damping:

$$x'(t'') = 1 - \cos(t'').$$

2. For $0 < \zeta < 1$, underdamped:

$$x'(t'') = 1 - \frac{e^{-\zeta t''}}{(1 - \zeta^2)^{1/2}} \cos [(1 - \zeta^2)^{1/2} t'' - \theta],$$

where $\theta = \cos^{-1}[(1 - \zeta^2)^{1/2}]$.

3. For $\zeta = 1$, critically damped:

$$x'(t'') = 1 - e^{-t''} (t'' + 1).$$

4. For $\zeta > 1$ (so that $\nu > 1$ also), overdamped:

$$x'(t'') = 1 + \frac{1}{(\nu - 1)} e^{-\nu^{1/2} t''} - \frac{\nu}{(\nu - 1)} e^{-\nu^{-1/2} t''}.$$

Typical response curves illustrating these four solutions are shown in Fig. 2-11.

Important points to note in this figure are listed below.

1. For all $\zeta > 0$, the final state is one of constant value equal to the input.

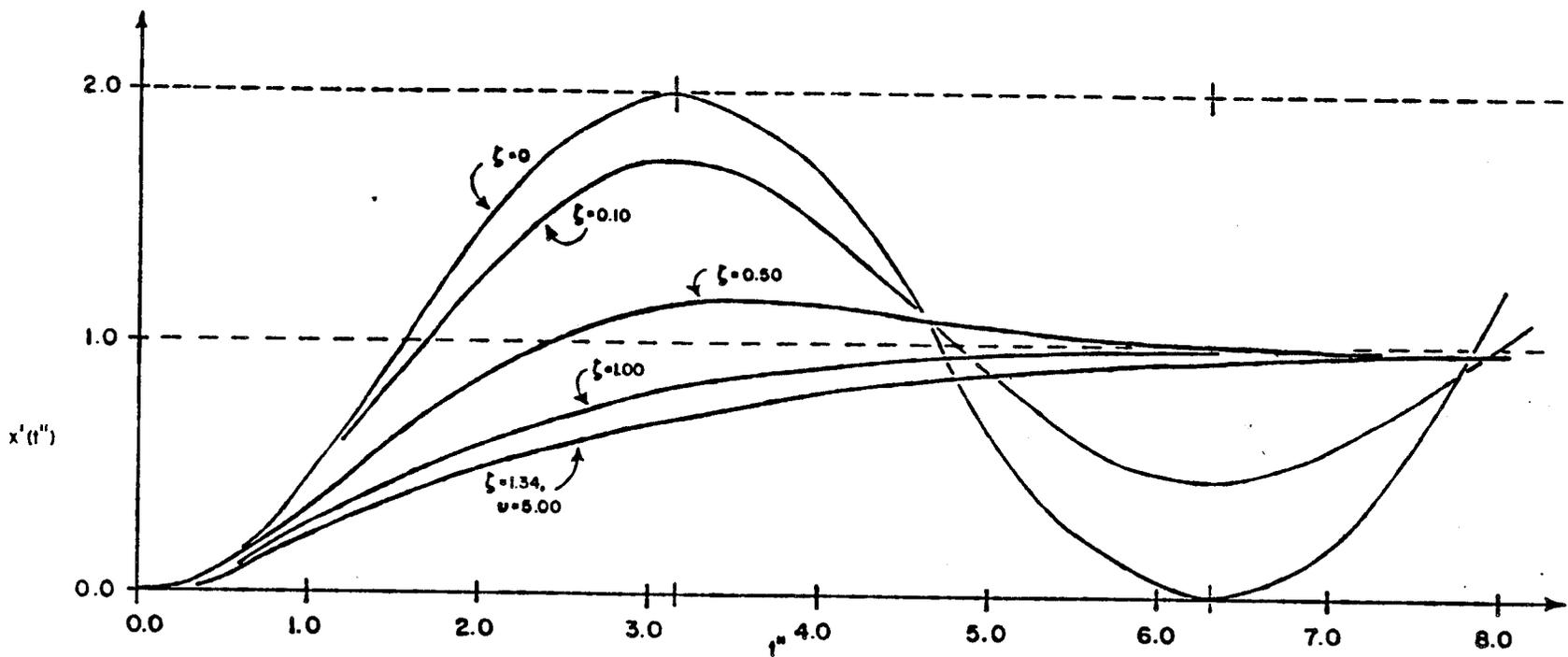
2. The slope of the response curve is continuous across $t'' = 0$. That the slope has the value zero at this point is the consequence of the imposed initial condition on dx/dt . However, the observed continuity is inherent and would be present even if a nonzero initial condition on the slope were specified. This continuity results from the "inertia" present in a second-order system. That is, the system contains a mass-equivalent component or element. Such an element cannot respond instantaneously to a change in the input.

3. In the limit where $\zeta = 0$, the system oscillates freely at a unit (dimensionless) frequency (corresponding to an angular frequency of ω_n). This motivated the definition of ω_n as the "natural frequency."

4. For each value of ζ , $0 < \zeta < 1.0$, the system undergoes (damped) oscillations at a dimensionless frequency

$$(1 - \zeta^2)^{1/2}.$$

Fig. 2-11. Step function response of a second-order system.



This corresponds to the previously defined (dimensional) angular frequency ω_m ; this result motivated the definition of ω_m as the modified (damped) natural frequency.

5. For small values of ζ , there is appreciable initial overshoot, and the oscillations last for a long time. As ζ is increased, the amplitude of the initial overshoot decreases as does the duration of the period of significant oscillations. The extrema or the oscillations in this case are at

$$t''_{en} = \frac{n\pi}{(1 - \zeta^2)^{1/2}}, \quad n = 1, 2, 3 \dots$$

Further, these extrema of the underdamped oscillations all lie on the two envelop curves given by

$$x'_e(t'') = 1 \pm e^{-\zeta t''}.$$

The amplitude of the oscillations exponentially decreases with time with a time constant of ζ^{-1} .

Ramp Input.

Teaching Note: For consistency with the notation used here, the slope of the ramp input, in dimensional terms, is taken to be

$$(x_\infty - x_0)\omega_n.$$

In dimensionless normalized form, the response to a ramp input is:

1. For $\zeta = 0$, no damping:

$$x'(t'') = t'' + \cos\left(t'' + \frac{\pi}{2}\right).$$

2. For $0 < \zeta < 1$, underdamped:

$$x'(t'') = (t'' - 2\zeta) + \frac{e^{-\zeta t''}}{(1 - \zeta^2)^{1/2}} \cos[(1 - \zeta^2)^{1/2} t'' + \theta],$$

where $\theta = \cos^{-1}[2\zeta(1 - \zeta^2)^{1/2}]$.

3. For $\zeta = 1$, critical damped:

$$x'(t'') = (t'' - 2)(1 - e^{-t''}).$$

4. For $\zeta > 1$ (so that $\nu > 1$ also), overdamped:

$$x'(t'') = \left[t'' - \frac{(\nu + 1)}{\nu} \right] - \frac{1}{\nu(\nu - 1)} (e^{-\nu^{1/2}t''} - \nu^2 e^{-\nu^{-1/2}t''}).$$

Important points to note in these response functions include

1. The new state (for $t'' \gg 1$) is one with uniform rate of change of output with time. The time-rate-of-change of the output is the same as that of the input.

2. At any instant, the output magnitude (for $t'' \gg 1$) is less than the input by an amount directly proportional to the damping ratio (damping number). This corresponds to a dimensional, nonnormalized difference in magnitude between input and output of $2\zeta(x_\infty - x_0)$.

3. The final response (for $t'' \gg 1$) lags the input by an amount directly proportional to the damping ratio (damping number). In a dimensional sense, this means that the output takes on a particular value at $2\zeta/\omega_n$ units of time after the input had that same value.

4. For small values of damping ratio, there is significant overshoot and continuing oscillations for an extended period.

Dynamic Characteristics, III

Settling Time. As defined in Section 2.2.2, the settling time t_s is a measure of the speed of response of an instrument, being the period of time in which the transient term is significant following a change in the input. Inspection of the responses to step and ramp inputs given above and recollection of the nondimensionalization of time with the natural frequency indicate that for a second-order system the settling time is a function of both the natural frequency and the damping ratio (or damping number). The dependence of t_s on ω_n is a simple inverse relationship; ω_n can itself be used as a measure of the speed of response of a second-order system.

Unfortunately, the dependency of t_s on ζ (or ν) is much more complicated. The complexity arises from the fact that both the duration and the magnitude (amplitude) of the transient are functions of ω_n . Examination of response curves of the type shown in Fig. 2-11 indicate qualitatively that an increase in ζ reduces both the rate and the amplitude of the oscillations for an underdamped system; more subtly, an increase in ζ also retards the rate at which the response approaches its final value. For a given value of natural frequency, the optimum value of damping ratio (in the sense of giving minimum settling time) will depend on the chosen tolerance band.

Lag. The concept of lag is again most applicable to the response to the ramp input. The ramp response functions show that in general

$$t''_1 = 2\zeta.$$

This result can be interpreted to indicate the more sluggish the system (large damping ratio/number, small natural frequency) the greater the lag.

As noted in Section 2.2.2, the concept of lag does not apply to a step input as the final state value eventually equals the input value.

Dynamic Error. Both the step response and the ramp response of a second-order system show transient dynamic errors. For small values of ζ , these can be quite large. However, these become negligible within the settling time as the final state is approached. Further, and more importantly, the response to a ramp input may show significant steady-state dynamic error. Inspection of the ramp response functions shows that

$$\lim_{t'' \rightarrow \infty} \epsilon_D = 2\zeta.$$

This result implies that use of small values of damping would be desirable if low steady-state dynamic error were the main criterion for evaluating a particular system.

Overshoot. In describing underdamped systems, the amplitude of the first overshooting peak, the largest, is often of interest. For a step input, the magnitude of this first peak is given by

$$x'(t''_{e1}) = 1 + e^{-\zeta t''_{e1}},$$

where

$$t''_{e1} = \frac{\pi}{(1 - \zeta^2)^{1/2}}.$$

Teaching Note. The response to the sharp leading edge (one with an instantaneous change) is in a sense a "worst case." While real world inputs may change abruptly, they never do so instantaneously. Consequently, overshoots encountered in practice will always be something less than the above value. Systems that might be rejected because of too large an overshoot to a step input may respond acceptably to real inputs.

Speed of Response. A distinct advantage of second-order systems is that they can be made to respond much faster to rapid changes in the input signal than a first-order system. However, a directly related disadvantage is that very rapid response is accompanied by undesirable overshooting oscillations or "ringing." The design of a second-order system invariably involves a tradeoff between the desired speed of response, and the amplitude of the initial overshoot acceptable to the purpose at hand.

Frequency Response

Sinusoidal Input. We consider the response of a second-order system to a sinusoidal input. For purpose of this analysis, the initial state of the system is again taken to be a steady one in equilibrium with the input. That is,

$$x(t) = x_I(t) = x_0, \quad t < 0,$$

and

$$\left. \frac{dx}{dt} \right|_0 = \left. \frac{dx_I}{dt} \right|_0 = 0, \quad t < 0.$$

We will assume the same type of sinusoidal input as used with the the first-order system in Section 2.2.2; the functional form will not be repeated here.

For present purposes, the transient phase of the response can be neglected and attention focused on the steady-state solution. The functional form of the steady-state portion of the response is the same regardless of the value of the damping ratio. With these remarks in mind, consider that

$$\lim_{t'' \rightarrow \infty} x'(t'') = x'_p(t''),$$

so that for the final, steady-state response to a sinusoidal input we obtain

$$x'(t'') = A(\beta) \cos [\beta t'' - \phi(\beta)].$$

Here β = dimensionless frequency (sometimes called the frequency number),

$$= \frac{\omega_I}{\omega_n},$$

$A(\beta)$ = normalized amplitude function (commonly called the amplitude ratio as it can be interpreted to be the ratio of the amplitude of the output signal to the amplitude of the input signal),

$$= \frac{1}{[(1 - \beta^2)^2 + (2\zeta\beta)^2]^{1/2}},$$

$\phi(\beta)$ = phase angle function,

$$= \tan^{-1}\left[\frac{2\zeta\beta}{1 - \beta^2}\right].$$

As before, the system response lags behind the input signal. This is indicated by the appearance of a minus sign in the argument of the sine function in the functional form of the response given above. The behavior of the system is described by the two functions $A(\beta)$ and $\phi(\beta)$; plots of these are shown in Figs. 2-12 and 2-13.

Recalling the comments made in Section 2.2.2 concerning ideal frequency response, consideration of the amplitude and phase angle functions given above shows that there is an optimum range of values for the damping ratio. The amplitude response for values of $0.6 < \zeta < 0.7$ are approximately flat over the widest range of frequency. Further, the corresponding phase angle curves are nearly linear over the full range $0 < \beta < 1.0$.

Dynamic Characteristics, IV

Resonance. One of the characteristic features of the frequency response of an underdamped second-order system is that for values of damping ratio smaller than a critical value the amplitude response function can be larger than unity over a broad range of frequencies. Thus a measuring system with second-order behavior can produce an output signal with an amplitude larger than that of the input, a marked difference from systems with first-order response. More specifically,

$$A(\beta) > 1,$$

for $0 < \beta < (2 - 4\zeta^2)^{1/2},$

when $0 < \zeta < 2^{-1/2}.$

Further, at the dimensionless frequency

$$\beta_p = (1 - 2\zeta^2)^{1/2},$$

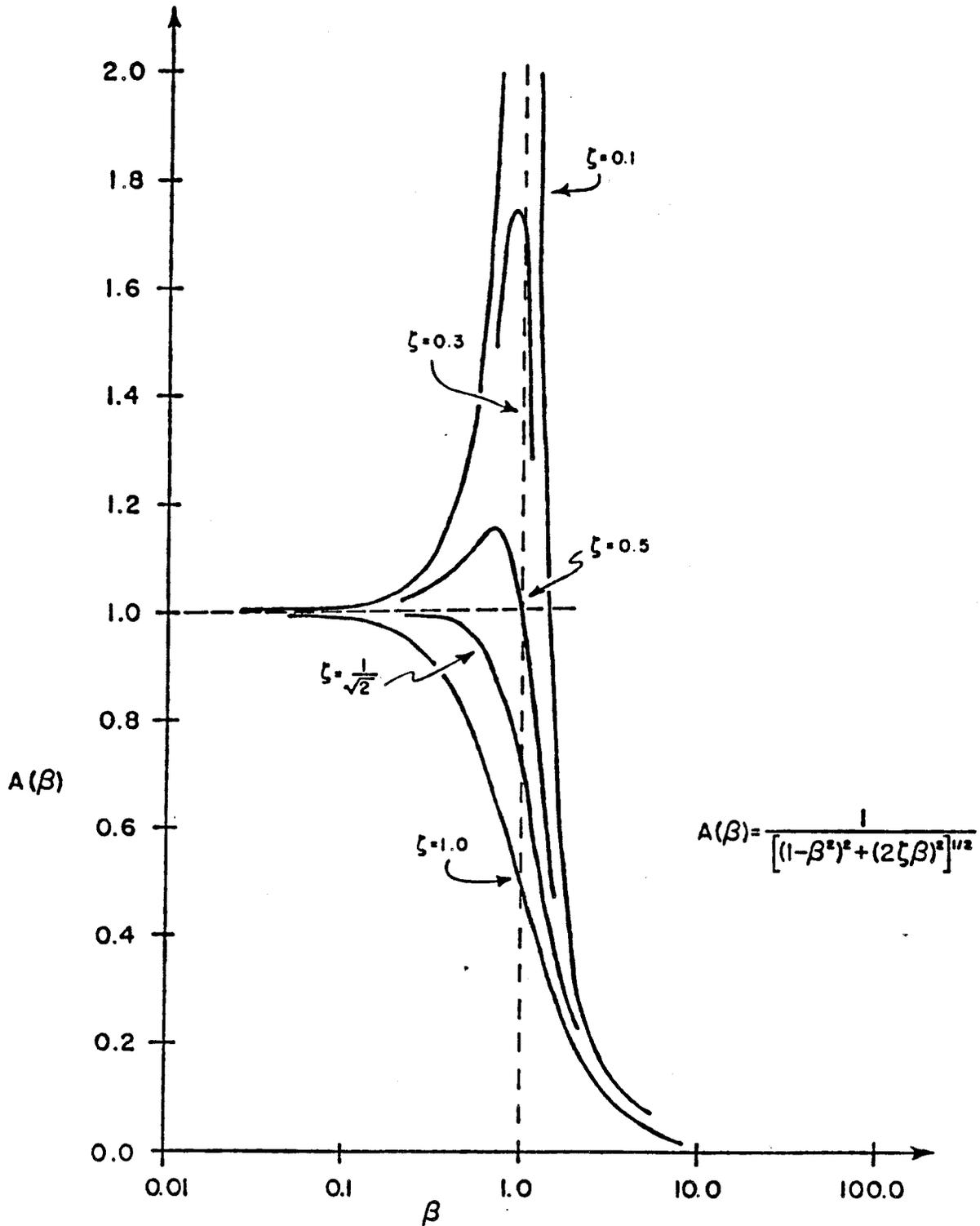


Fig. 2-12. Amplitude function of a second-order response to a sinusoidal input.

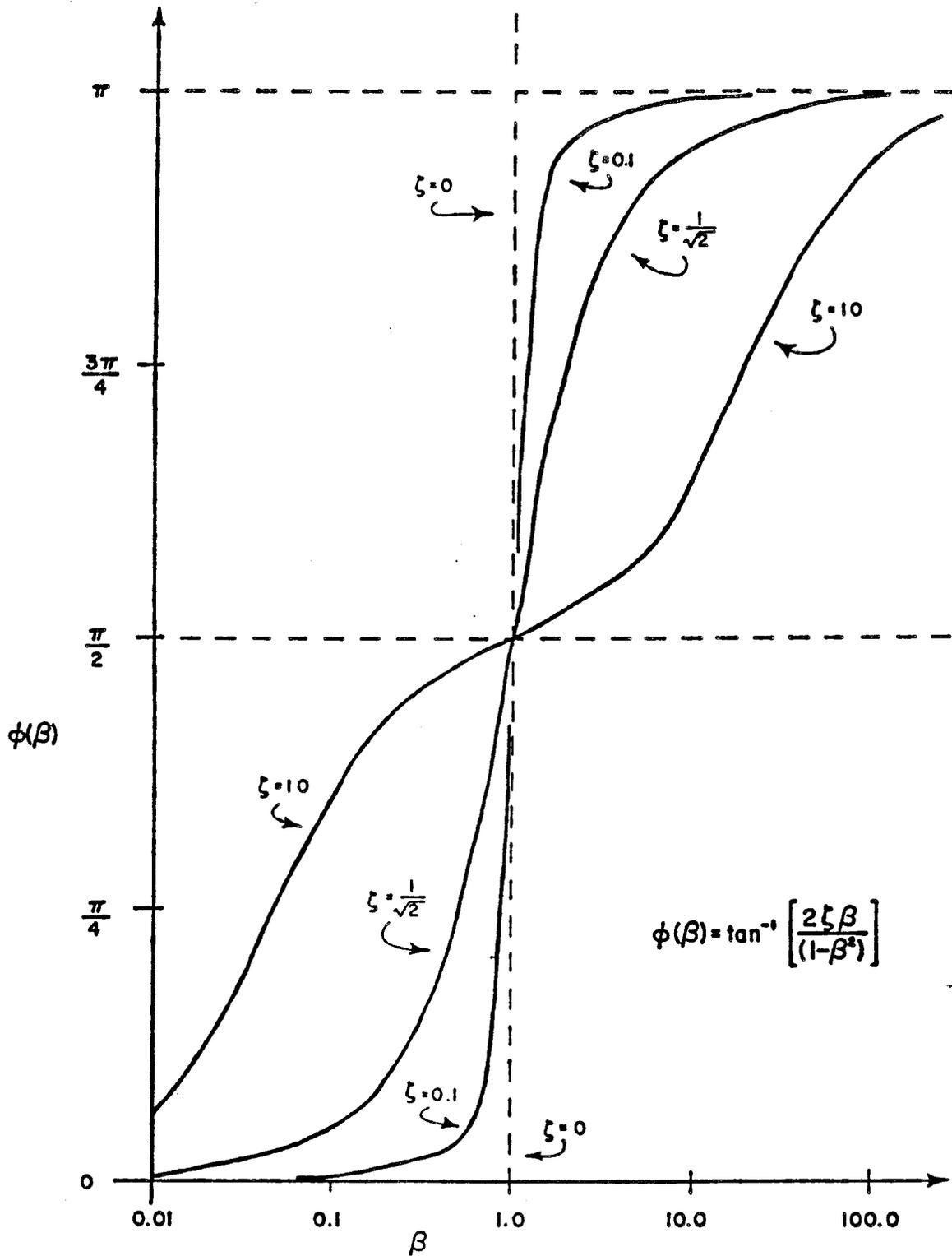


Fig. 2-13. Phase function of a second-order response to a sinusoidal input.

the amplitude response will have a maximum or peak value of

$$A(\beta_p) = \frac{1}{2\zeta(1 - \zeta^2)^{1/2}}$$

with corresponding phase angle

$$\phi(\beta_p) = \tan^{-1} \left[\frac{(1 - 2\zeta^2)^{1/2}}{\zeta^2} \right].$$

Note that for small values of damping, the peak amplitude ratio $A(\beta_p)$ can become very large.

This phenomenon of a local peak of value greater than unity in the amplitude response curve is termed resonance. The local maximum is called the "resonant peak" and β_p the "resonant frequency."

Resonance is found to occur in weakly damped systems containing two different types of energy-storage elements.

The width of a resonant peak in an amplitude response curve is conventionally given in terms of the separation (in frequency) of the $\pi/4$ - and $3\pi/4$ -points on the phase curve:

$$\text{-- for the } \frac{\pi}{4} \text{-point: } \beta_l = -\zeta + (\zeta^2 + 1)^{1/2},$$

$$\text{-- for the } \frac{3\pi}{4} \text{-point: } \beta_u = +\zeta + (\zeta^2 + 1)^{1/2}.$$

Consequently, the peak always has a width of 2ζ . In general, it is not symmetrical about β_p . These features are illustrated in the schematic amplitude and phase curves shown in Fig. 2-14.

Teaching Note. An element of confusion is often encountered in descriptions of resonance and resonant effects. This occurs because in many applications of second-order systems, it is desired to make $A(\beta_p)$ as large as possible (which also makes the peak very narrow) by making the damping ratio small. Consequently, the following approximate forms are frequently encountered in textbooks:

1. Resonant frequency: $\beta_p = 1,$

2. Amplitude of the resonant peak: $A(\beta_p) = \frac{1}{2\zeta} \gg 1,$

3. Phase angle of the resonant peak: $\phi(\beta_p) = \frac{\pi}{2},$

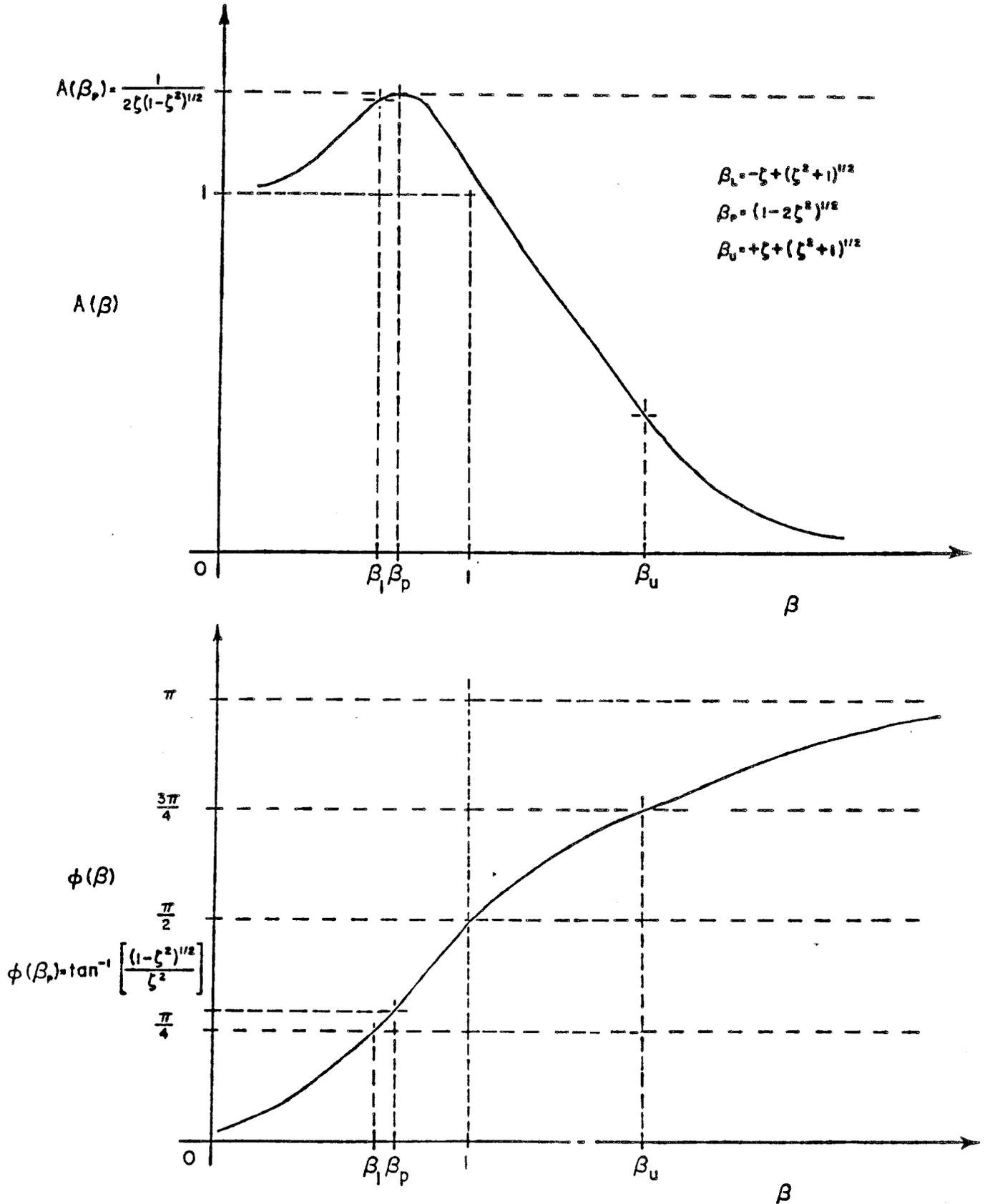


Fig. 2-14. Resonant peak amplitude and phase response of a second-order system to a sinusoidal input.

$$4. \quad \frac{\pi}{4} \text{-point: } \beta_1 = 1 - \zeta,$$

$$5. \quad \frac{3\pi}{4} \text{-point: } \beta_u = 1 + \zeta.$$

Also, in this case

$$A(\beta_1) = A(\beta_u) = \frac{1}{2\zeta(2)^{1/2}}.$$

In this approximate formulation, the resonant peak is taken to be symmetrical in shape and centered at the natural frequency; its width is unchanged at $\beta_u - \beta_1 = 2\zeta$.

Recall that the power contained in a signal is proportional to the square of the signal amplitude. Thus the output power will be proportional to the square of the amplitude function. The β_u and β_1 points are in this approximate formulation the "half-power points" bracketing the central peak.

It should be carefully pointed out to the student that these expressions are approximations valid only for very small ζ . This is seldom done; in many texts one finds statements such as "resonance occurs when the driving frequency equals the natural frequency" and "the amplitude of the resonant peak is $1/2\zeta$." Such statements are true only for small damping ratios. This point is particularly confusing to many students because most second-order systems of meteorological interest have damping ratios around 0.6 to 0.7, too large for the approximate forms to be used.

The resonant peak in the amplitude response appears only for $\zeta < 2^{-1/2}$. On a plot of the amplitude response versus dimensionless frequency, the curve for $\zeta = 2^{-1/2}$ is thus a dividing line between resonant and nonresonant response. Note that some underdamped systems (in a time response sense), ones with

$$2^{-1/2} < \zeta < 1.0,$$

are not resonant.

Roll-Off. For $\zeta = 2^{-1/2}$, the amplitude response function becomes

$$A(\beta) = \frac{1}{(1 + \beta^4)^{1/2}}.$$

This form is similar to that of the amplitude response function for a first-order system but with a fourth-power dependence (instead of a square) on the dimensionless frequency. Consequently this amplitude function is "flatter" at the lower frequencies and "rolls-off" quicker at the higher frequencies (i.e., has a steeper downward slope to the amplitude response curve).

2.2.4 EXAMPLE OF A FIRST-ORDER SYSTEM

The first-order system can be represented by

$$a_1 \frac{dx}{dt} + a_0 x = b_0 x_i. \quad (1)$$

Three parameters a_1 , a_0 , and b_0 are used but only two are necessary because one of the above can be reduced to unity by dividing through equation (1) by, say a_0 . We shall write the first-order equation in standard form

$$\frac{\tau dx}{dt} + x = K_s x_i,$$

or

$$\frac{x}{x_i} = \frac{K_s}{\tau p + 1}, \quad (2)$$

where

$$K_s = \frac{b_0}{a_0} = \text{static sensitivity with the dimensions of input/output, and}$$

$$\tau = \frac{a_1}{a_0} = \text{time constant with the dimension of time.}$$

$$p = \frac{d}{dt} = \text{derivative operator.}$$

The liquid-in-glass thermometer Fig. 2-15 is an example of a first-order system. The thermometer is exposed to air of temperature $T_i(t)$ which is the input quantity usually designated as $x_i(t)$. The output is the height, x , of the column relative to some arbitrary reference point here designated as the point $x = 0$ which corresponds to the equilibrium of temperature of 0°C . A tube with a fine bore, called a stem, is attached to the bulb holding the liquid, and the mass of liquid in the thermometer is adjusted so that the bulb is filled completely but the stem is only partially filled at all temperatures in

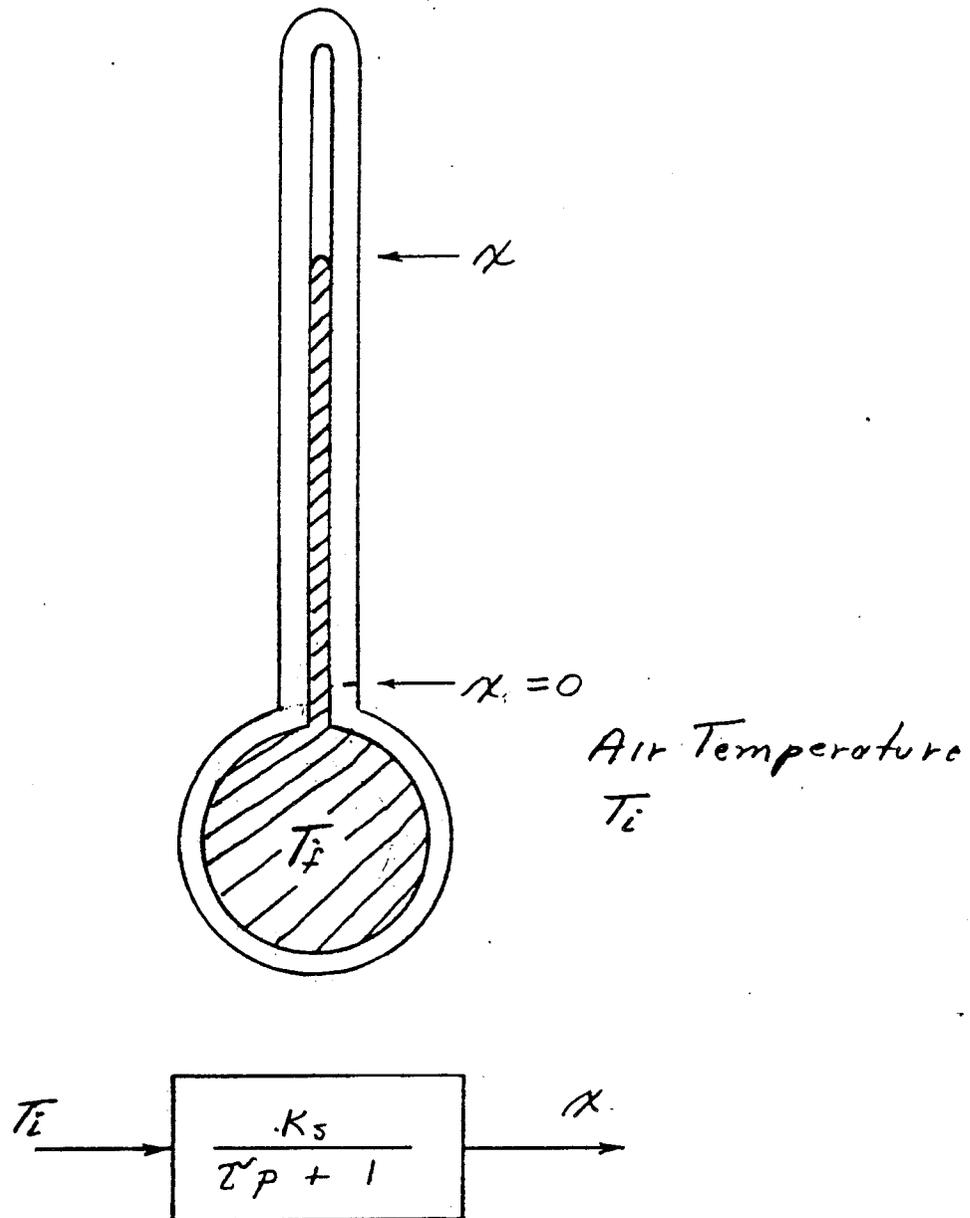


Fig. 2-15. Liquid-in-glass thermometer system.

the specified range of the thermometer. The height of the column, x , is related to the temperature of the liquid by

$$x = \frac{K_{ex} V_b}{A_c} T_f \quad (3)$$

Where

x = relative height of the column,

T_f = temperature of the liquid in the bulb (assumed uniform throughout),

K_{ex} = differential thermal expansion coefficient of liquid and glass,

V_b = volume of bulb,

A_c = cross-sectional area of capillary tube.

The differential equation relating input and output can be derived from considering conservation of energy for the thermometer bulb.

(heat in) - (heat out) = energy stored,

$$UA_b (T_i - T_f) dt = V_b \rho C dT_f, \quad (4)$$

where

U = overall heat-transfer coefficient across the bulb wall,

A_b = area of bulb,

ρ = density of thermometer fluid,

C = specific heat of thermometer fluid.

Equation (4) involves many assumptions:

1. The bulb and stem walls have no heat storage capacity. This assumption is not always satisfied, however, we usually ignore this problem.
2. The overall coefficient U is constant. It is a function of air density and wind speed. For wind speed in excess of 30 cm/s at normal density, the time constant for a mercury-in-glass thermometer should be on the order of one minute.

3. No heat is lost from the thermometer bulb by conduction up the stem. Ideally, the mounting support temperature will be close to air temperature.
4. No heat transfer by radiation.

This short list of some of the more important assumptions serves to emphasize that Eqs. (3) and (4) are not exact models for the thermometer behavior but do constitute a model which has proven to be sufficiently good for all but the most demanding applications. The adequacy of a model must always be judged in light of its applications and must be experimentally verified.

Combining Eqs. (3) and (4), we get

$$\frac{\rho C A_c}{K_{ex}} \frac{dx}{dt} + \frac{U A_b A_c}{K_{ex} V_b} x = U A_b T_i, \quad (5)$$

and by comparison with Eq. (2), we define

$$K_s = \frac{K_{ex} V_b}{A_c},$$

$$\tau = \frac{\rho C V_b}{U A_b}.$$

Because it is common to introduce the refinement of putting a scale on the thermometer in temperature units, we can set $K_s = 1$ and get, as the final performance equation:

$$\tau \frac{dT}{dt} + T = T_i, \quad (6)$$

where T = output temperature as indicated by the attached scale.

2.2.5 NEED FOR HIGHER ORDER AND NONLINEAR MODELS

In order to apply the first- and second-order differential equations used above a number of simplifying assumptions have to be made concerning the instruments. No real instrument will exactly satisfy these assumptions. The first- and second-order responses discussed above are thus only approximations to real instrument behavior.

As an example of the complexity involved in a real instrument, consider the development in the example section above. The behavior of a liquid-in-glass thermometer was shown to be approximately described by a first-order differential equation. This was done by considering only the heat storage in the liquid in the bulb but neglecting heat storage in the glass bulb, glass stem, and in the liquid in the column. When these are considered, there are four energy reservoirs. A full description of the thermometer thus requires a fourth-order differential equation. An appreciation of the complexity of the response which results can be gained by noting that, when a thermometer is plunged into a warm environment, the level of the liquid in the column can actually momentarily fall before beginning to rise. This is caused by the expansion of the bulb.

Another simplifying assumption is that an instrument can be modeled by a linear differential equation with constant coefficients. In the case of the liquid in glass thermometer, the assumption was made that the thermal expansion coefficient of the liquid is independent of temperature. This is only approximately true.

In this section, we briefly discuss the governing equations for systems which are higher than second order and possibly nonlinear. One key result is that while all systems are to some degree higher order and nonlinear many systems can be approximated as first- or second-order linear. Even for systems which are inherently of higher than second order the essential behavior can be understood in terms of performance characteristics which are natural extensions of the characteristics of first- and second-order systems.

Higher Order Equations

For a thorough discussion of higher order differential equations and how they apply to meteorological instruments see Bottaccini (1975, Chaps. 2 and 4). A text on ordinary differential equations will also be helpful.

One possible way for such equations to arise is by connecting simpler systems in series or by having a system in which the various components are coupled. A good example is a propeller vane. The vane is approximately second-order; however, rotation of the vane affects the propeller so that the indicated wind speed is described by a third-order equation.

A general nth-order differential equation is

$$a_n \frac{d^n x}{dt^n} + a_{n-1} \frac{d^{n-1} x}{dt^{n-1}} + \dots + a_1 \frac{dx}{dt} + a_0 x = b_0 x_i \quad (7)$$

where

$x_i(t)$ = input
 $x(t)$ = output
 t = time
 a 's, b 's = system physical parameters, usually assumed to be constant.

The term "system" refers to a real measurement system or to our idealized concept of it. When a system is described as being, say, second-order, linear, one means that it is reasonable, within limits, to use a second-order, linear ordinary differential equation as a model to describe its dynamic performance.

A system or an equation is linear if the superposition principle holds, i.e., if input $x_i(t)$ produces output $x(t)$ then the input

$$c_1 x_{i1}(t) + c_2 x_{i2}(t)$$

produces the output

$$c_1 x_1(t) + c_2 x_2(t).$$

A system is time-invariant (the corresponding differential equation has constant coefficients) if when input $x_i(t)$ produces output $x(t)$, then the input $x_i(t + t_0)$ produces the output $x(t + t_0)$. In other words, the dynamic response is not a function of when the input was applied.

Operational Transfer Function

Define the operator p such that

$$p x(t) = \frac{dx(t)}{dt}$$

$$p^2 x(t) = \frac{d^2x(t)}{dt^2},$$

Then Eq. (7) can be written as

$$(a_n p^n + a_{n-1} p^{n-1} + \dots + a_1 p + a_0) x = b_0 x_i \quad (8)$$

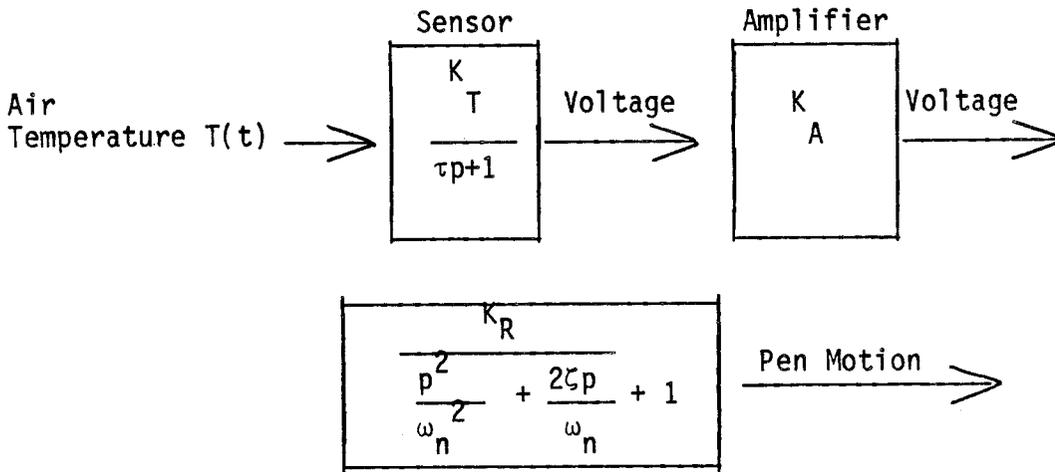
or by algebraic manipulation,

$$x(t) = \frac{b_0}{a_n p^n + a_{n-1} p^{n-1} + \dots + a_1 p + a_0} x_i(t) \quad (9)$$

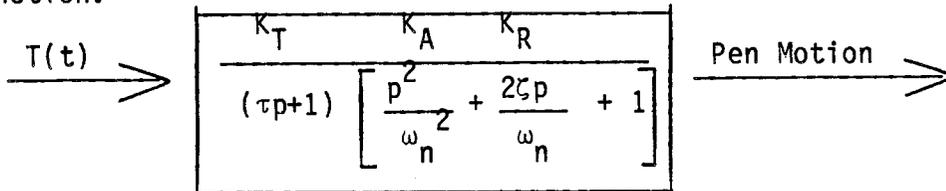
$$= H(p) x_i(t).$$

Then $H(p)$ is called the operational transfer function. The transfer function facilitates the use of block diagrams of systems.

Example: Air temperature recording system in block diagram and operator notation.



This system can also be represented with a single transfer function.



Relation to Laplace and Fourier Transfer Functions

The Laplace transfer function is the ratio of the Laplace transform of the output to the Laplace transform of the input when all initial conditions are zero. The Laplace transform is

$$L [x(t)] = X(s) = \int_0^{\infty} x(t)e^{-st} dt \quad (10)$$

and the corresponding transfer function is

$$\frac{X(s)}{X_i(s)} = \frac{b_0}{a_n s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0} = H(s) \quad (11)$$

where $s = \sigma + j\omega$ is a complex variable. The Fourier transform

$$F[x(t)] = X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt \quad (12)$$

and the transfer function is

$$\frac{X(j\omega)}{X(j\omega)} = \frac{b_0}{a_n (j\omega)^n + a_{n-1} (j\omega)^{n-1} + \dots + a_1 (j\omega) + a_0} = H(j\omega) \quad (13)$$

where $j = \sqrt{-1}$ and $\omega =$ frequency in radians/second.

The similarity of Eqs. (9), (11), and (13) suggest that one can readily convert between operator- p notation, Laplace s notation and Fourier ($j\omega$) notation and that one can represent the transfer function as $H(p)$, $H(s)$, or $H(j\omega)$. This provides a simple means for transferring from the time domain, operator- p notation, to the frequency domain, Fourier ($j\omega$) notation. The property of the transfer function to map the system described by a linear ordinary differential equation into the frequency domain is one of the most powerful tools available for system analysis.

Complete Solution of a Differential Equation

The complete solution of Eq. (8) is

$$x(t) = x_t(t) + x_s(t)$$

where $x_t(t)$ is the transient response and $x_s(t)$ is the steady-state response. The transient response is that part of the total response which is a function of initial conditions. If the system described by Eq. (8) is stable, the transient response decays and goes to zero as t approaches infinity. The steady-state response is independent of initial conditions.

The transient response function has n arbitrary constants determined by the initial conditions. The form of this response is determined by the characteristic equation, obtained by replacing the operator- p with the algebraic variable, r ,

$$a_n r^n + a_{n-1} r^{n-1} + \dots + a_0 = 0 \quad (14)$$

When the roots r_1, r_2, \dots, r_n have been found, the form of the transient solution is given by the following rules:

1. Real roots, unrepeated. For each real unrepeated root, r , there will be a ce^{rt} term in the solution. The constant c is determined from the initial conditions.
2. Real roots, repeated. For each real root, r , repeated m times, the terms in the solution will be

$$(c_0 + c_1 t + c_2 t^2 + \dots + c_{m-1} t^{m-1}) e^{rt}.$$

3. Complex roots, unrepeated. Complex roots have the form of $a + jb$, and if the coefficients in Eq. (7) are real, complex roots will occur in the conjugate pairs, $a + jb$. For each such pair, the corresponding term in the solution will be

$$ce^{at} \cos(bt + \phi) = e^{at}(c_1 \cos bt + c_2 \sin bt),$$

where $c = \sqrt{c_1^2 + c_2^2}$ and $\phi = \tan^{-1}(-c_2/c_1)$.

4. Complex roots, repeated. For each pair of complex roots which appears m times, the solution is

$$c_0 e^{at} \cos(bt + \phi_0) + c_1 t e^{at} \cos(bt + \phi_1) + \dots \\ + c_{m-1} t^{m-1} e^{at} \cos(bt + \phi_{m-1}).$$

The transient solution is simply the sum of the individual terms. To evaluate the constants, c_i , there must be n initial conditions specified.

The steady-state solution can be found by the method of undetermined coefficients. Given that the input is some function of $x_i(t)$, repeatedly differentiate $x_i(t)$ until the derivatives go to zero or repeat the functional form of some lower-order derivative. This is also the test for the applicability of the method: if neither of the above conditions prevail, the method of undetermined coefficients cannot be used. Write the steady-state solution as

$$x_s(t) = k_1 x_i(t) + k_2 p x_i(t) + k_3 p^2 x_i(t) + \dots \quad (15)$$

where the right-hand side includes one term for each functionally different form found by examining $x_i(t)$, and its derivatives. The

constants k_i do not depend upon the initial conditions. They are found by substituting Eq. (15) into Eq. (7).

The Sinusoidal Transfer Function

It is useful to study the response of measurement systems to standard inputs. The most useful response is the steady-state response to a sinusoidal input. The input is $x_i(t) = A_i \sin \omega t$ and the response, after the transient dies out, will be $x(t) = A_o \sin(\omega t + \phi)$. The output will have the same wave form and the same frequency as the input but usually a different amplitude, and there will be some phase shift ϕ .

Therefore, the system frequency response can be specified in terms of an amplitude ratio and a phase shift, both functions of frequency.

It is convenient to find the sinusoidal response by first getting the response to the input

$$x_i(t) = A_i e^{j\omega t}$$

where $A_i e^{j\omega t} = A_i (\cos \omega t + j \sin \omega t)$.

Assume that the steady-state output will be $x(t) = c e^{j\omega t}$ where c is complex and a function of ω . To find c , substitute the assumed solution into Eq. (8) noting that

$$p^m x = c(j\omega)^m e^{j\omega t}$$

so we get

$$(a_n (j\omega)^n + a_{n-1} (j\omega)^{n-1} + \dots + a_1 (j\omega) + a_0) c e^{j\omega t} = b_0 a_i e^{j\omega t}$$

and

$$c/A_i = \frac{b_0}{a_n (j\omega)^n + a_{n-1} (j\omega)^{n-1} + \dots + a_1 (j\omega) + a_0} = H(j\omega). \quad (16)$$

The sinusoidal steady-state response for a linear system can be deduced from the response to the exponential input using the super-position principle.

$$\cos \omega t = \text{Re}[e^{j\omega t}]$$

$$\sin \omega t = \text{Im}[e^{j\omega t}]$$

where Re denotes "real part of" and Im denotes "imaginary part of."

$$\begin{aligned}
 \text{When } X_i(t) &= A_i e^{j\omega t}, \\
 X(t) &= H(j\omega) A_i e^{j\omega t} \\
 &= |H(j\omega)| A_i e^{j\omega t} e^{j\phi} \\
 &= |H(j\omega)| A_i e^{j(\omega t + \phi)}.
 \end{aligned}$$

$$\text{Note that } H(j\omega) = |H(j\omega)| e^{j\phi}$$

$$\text{where } |H(j\omega)| = \sqrt{\text{Re}[H(j\omega)]^2 + \text{Im}[H(j\omega)]^2}$$

$$\text{and } \phi = \tan^{-1} \left[\frac{\text{Im}[H(j\omega)]}{\text{Re}[H(j\omega)]} \right].$$

When $X_i(t) = A_i \cos \omega t$, the output will be the real part of the above solution so

$$\begin{aligned}
 X(t) &= |H(j\omega)| A_i \text{Re} \left\{ e^{j(\omega t + \phi)} \right\} \\
 &= |H(j\omega)| A_i \cos (\omega t + \phi) \\
 &= A_0 \cos (\omega t + \phi).
 \end{aligned}$$

and when $X_i(t) = A_i \sin \omega t$, the output will be the imaginary part so

$$\begin{aligned}
 X(t) &= |H(j\omega)| A_i \text{Im} \left\{ e^{j(\omega t + \phi)} \right\} \\
 &= |H(j\omega)| A_i \sin (\omega t + \phi) \\
 &= A_0 \sin (\omega t + \phi).
 \end{aligned}$$

For example, for a first-order system

$$H(j\omega) = (1 + j\tau\omega)^{-1}$$

$$\text{Re}[H(j\omega)] = (1 + (\tau\omega)^2)^{-1}$$

$$\text{Im}[H(j\omega)] = -\tau\omega(1 + (\tau\omega)^2)^{-1}$$

$$\text{so } |H(j\omega)| = \frac{1}{\sqrt{1 + (\tau\omega)^2}}$$

$$\phi = \tan^{-1}(-\tau\omega).$$

If the input to a first-order system is $X_i(t) = A_i \sin \omega t$, the steady-state output will be

$$X(t) = \frac{A_i}{\sqrt{1 + (\tau\omega)^2}} \sin(\omega t + \phi)$$

which was shown above. We then see that the response of the n th-order system to sinusoidal forcing is sinusoidal with the same frequency as the forcing, that the response has an amplitude given by $|H(j\omega)|$ and that the response lags behind the forcing function by a phase angle ϕ . This is obviously a simple generalization of the response of a second-order system.

The value of $|H(j\omega)|$ is a function of frequency called the "response function" or "filter function" of the instrument. This function gives the amplitude of the response of the instrument as a function of forcing frequency. Because the response function is the inverse of an n th-order polynomial, the curve can be much more complicated than the response curves of a second-order system; however, this response function can exhibit properties such as resonance which are similar to those of second-order systems. One very useful property of the response function is that it can readily be used to correct the spectrum of measured quantities.

Note that the most general solution to the differential equation requires specification of initial conditions. This general solution is the superposition of the steady-state solution just derived with transient solutions which are simply the responses to step function input as given above. Because this is a linear differential equation with constant coefficients, we may obtain solutions for general forcing by decomposing the forcing into its Fourier components. The response of the system is then the linear superposition of the responses to the individual Fourier components.

Nonlinear Systems

As discussed above, real instruments are to some degree nonlinear and have coefficients which are functions of time. Solutions to such differential equations are much more difficult than solutions to linear constant coefficient equations, and no general forms of solution can be

found. Particularly troublesome properties of such equations are that the forcing and response frequencies are no longer the same and that one can no longer use the powerful properties of linear superposition of solutions and use of Fourier decomposition of the forcing to obtain solutions. For these reasons we generally avoid using nonlinear or nonconstant coefficient differential equations to model instrument behavior.

Reduction to Lower Order Systems

Returning to the discussion of the liquid-in-glass thermometer above, remember that we said that a close look at the thermometer reveals that it really has four energy reservoirs. These reservoirs do not all contain the same amount of energy, and the influence of these reservoirs on instrument response is not equal. Most of the energy in the thermometer is contained in the mercury in the bulb, and the temperature indicated by the thermometer is mostly determined by the temperature of the mercury in the bulb. For this reason, we can explain most of the behavior of the thermometer ignoring the other reservoirs and treating the thermometer as a first-order system. For most instruments there will be one or two dominant reservoirs and a first- or second-order differential equation can be used for all but the most detailed treatment of many instruments.

2.2.6 QUESTIONS, PROBLEMS, AND LABORATORY EXERCISES

Questions and Problems

1. In talking about dynamic characteristics we have disregarded Section 2.1. How will each of the static response characteristics affect the output of an instrument exposed to a time-varying atmosphere?
2. Examine the solution we obtained for the lag of an instrument which is exposed to sinusoidal forcing. Is it ever possible for the lag of the instrument to become negative? Give a physical interpretation of your answer.

Laboratory Exercises

Laboratory Exercises for Dynamic Response

Laboratory Exercise #1: Response to a Step Function (Manual)

Equipment:

A thermometer.
Two containers with water at different temperatures.
A watch which indicates seconds.

Instructions:

Students note the temperature of one bath and then plunge the thermometer into the other bath and record the temperatures indicated at various predetermined times. Repeat this several times alternating between the baths. Try it several times recording the times at which predetermined temperatures occur. Plot the results to obtain response times. Discuss the comparative accuracy of the two methods of taking measurements.

This experiment may be done in the wind tunnel at various wind speeds. Air at two different temperatures may also be used. For some thermometers this is necessary in order to get slow enough responses to manually read.

Instruments other than thermometers may also be used.

Laboratory Exercise #2: Response to a Step Function (Computer)

Equipment:

A computer having an analog to digital converter.
Any type of electrically indicating thermometer.

Method:

This experiment is done similarly to the manual response time experiment except that the computer is used.

Other Suggestions:

A number of problems and exercises may be given which illustrates the concepts of dynamic response. A computer can be used to simulate instruments with various characteristics which the students can specify for various forcing functions. They may plot graphs to see how their instruments will respond.

Another good possibility is for the student to specify a response time for the thermometer and hygrometer in a radiosonde and to see how the measured sounding of dewpoint and temperature compare to the actual sounding. This can either be done using a computer or by hand-computation.

The computer can also be used to simulate and plot the response of an unknown instrument. The student must then guess the order of the instrument and the values of the required constants and choose a plotting format to test the guesses. Further guesses may be made until good values are obtained.

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CHAPTER 3

CALIBRATION STANDARDS

Contributors:

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3.0 INTRODUCTION

The "fineness" required of meteorological instruments varies widely, depending on which variables and applications we are concerned with. We may, for example, want to know the pressure to one tenth of a millibar or one hundredth of one percent absolute (or 0.01% of range), temperature to one third of a degree or about one tenth of a percent absolute (or 0.3% of range), wind speed to say 0.2 mph or one percent absolute (or 1% of range), solar radiation to say 70 w/m² or to five percent absolute or relative humidity to 10 percent absolute. Before we discuss these limits further, we need to define what we mean by accuracy or bias.

The first step in determining the accuracy of an instrument is calibration--a comparison of the instrument against a standard of previously established accuracy. Ideally, the result of this exercise would give a quantitative idea of how closely the output of the instrument represents the true value of the quantity being measured. However, since perfect sensors do not exist, accuracy is generally determined relative to the best available standard.

The concept of calibration carries an important implication--that the instrument calibrated possesses long-term stability with respect to its period of intended use. Otherwise, the calibration is a futile exercise. At the very least, a record of the calibrations of a particular instrument--its calibration history--should show its drift over time.

Thus quality assurance begins and ends in a calibration laboratory or field installation where the calibration history of an instrument is established, allowing us to make claims as to its accuracy in the future, which is what we want.

In the laboratory we establish the bias or accuracy or offset of a sensor. This systematic error is then removed by the calibration process.

Imprecision describes the remaining random error which is nonrepeatable and cannot be removed but only defined by the calibration.

Since almost all meteorological sensors have nonlinear responses, additional errors are introduced if we approximate a response with, for example, a straight line. These nonlinearity errors are frequently neglected, but with computer processing this is not justified today. We also need to specify the range over which the calibration is valid.

Some sensors, such as anemometers, have lower response limits. These are called the starting thresholds or simply the thresholds. Though frequently not specified or taken into account, there is a significant region of nonlinear response above the threshold for which the linear calibration given for an instrument is not valid. Disregarding the threshold and the nonlinear response in the region above it, may result in significant errors.

The resolution, which is often confused with the accuracy of a sensor, is the smallest change of input which produces a definite change in output. Resolution errors are, for example, introduced when an instrument is read visually.

Some sensors exhibit an offset called hysteresis, which is a function of the direction and magnitude of the input change and is caused by friction.

The combined effect of the different random uncertainties may be expressed by the root sum square (rss) of individual uncertainties. For example, one source of uncertainty in calibrating a barometer may be 0.25 mb, and another is the process of reading it to one millibar. (Rounding is a random process.) This produces an rss value of

$$\sqrt{1^2+0.25^2} \quad \text{or}$$

1.06 mb for the total uncertainty.

By necessity when calibrating instruments in the controlled and confined environment of the laboratory, one must assume that, for the measured quantity, test conditions closely simulate the uncontrolled natural environment. While this is generally true for such meteorological variables as temperature and pressure, it is not true for other variables such as precipitation and wind with its natural turbulence.

Increased use of automated sensing systems has prompted the American Society for Testing and Materials (ASTM) to attempt to standardize calibrations or at least comparisons of such measurement systems. In their proposed guidelines (Section 3.4.1), operational comparability of meteorological measurements is defined by specified statistical parameters based on simultaneous measurements from the systems under comparison in their natural environment. The measurements are to be made within a specified volume (generally a 10-meter-diameter cylinder 1 meter high). The root mean square (rms) of the measurement pairs is referred to as the "operational comparability" for

different instruments and "functional precision" for identical instruments.

Efforts to maximize instrumentation accuracy and precision should always be weighed against a realistic assessment of the user's need for quality. It is important to understand for what purpose the data are to be collected. Very often we make measurements at one spot for a limited time, which we then assume to be representative of a larger area under average conditions. Since the spot measurement is only one sample of an infinite number of sites in the area and over an infinite time period, there is an inherent statistical uncertainty in the measured data when they are used to represent the ensemble average. This uncertainty, which is very difficult to quantify, is generally larger than the sensor or recording uncertainties (see Wyngaard, 1983). If several instruments are used for an area-wide comparison, it is more important to have a good relative calibration between instruments than absolute ones, as gradients are generally more important than the absolute magnitudes. If one of the instruments has an absolute calibration, its calibration may be transformed to other instruments through the relative calibrations.

3.1 STANDARDS AUTHORITIES

Several nations maintain standards laboratories for the purpose of making the best possible measurements of physical quantities. Sensors maintained by these organizations serve as standards against which other sensors may be calibrated. Of usefulness to the meteorological community are the National Bureau of Standards (NBS) in the United States, the Applied Physics Division of the Commonwealth Scientific and Industrial Research Organization (CSIRO) in Australia, the National Research Council (NRC) or the Atmospheric Environment Service (AES) in Canada, and the National Physical Laboratories (NPL) in the United Kingdom. Because these laboratories are independent, periodic international intercomparisons are held to assess the relative agreement of national standards. An exception is radiation for which one worldwide ultimate standard is maintained by the World Meteorological Observation in Switzerland.

The goal of standards laboratories is to make the very best possible measurement of physical quantities. The accuracies to which they aspire are generally, but not always (e.g., wind speed), orders of magnitude greater than are required in meteorological research. Thus a sensor for field work can be adequately calibrated with considerably greater inaccuracy than is tolerated in a standards laboratory. Nevertheless, it is useful to be aware of the ultimate calibration philosophy for measuring particular physical quantities.

3.2 TYPES OF STANDARDS

Primary Standard

There is a hierarchy of standards with decreasing precision. The highest standard is called the primary standard. It has two definitions:

- A standard calibrated with respect to the fundamental quantities, time mass and length; or
- A standard of highest attainable precision.

While the first definition is more attractive from a theoretical point of view, the second is more practical and less ambiguous. Primary standards are generally the concern of standard authorities, such as NBS where the precisions strived at and achieved are beyond what is required by the meteorological community.

Secondary Standard

The highest standards used by the meteorological community are referred to as secondary standards, defined as a standard whose calibration can be traced directly or indirectly to a primary standard. This trace to a primary standard is established by regularly sending the secondary standard to NBS for a comparison against their primary standards. NBS performs such services for a fee (NBS Special Publication No. 250) of typically a few hundred dollars per instrument. This secondary standard will then serve as the laboratory standard at individual institutes and universities against which other instruments are calibrated.

Reference Standard

The reference standard at a location is defined as the best standard locally available. A reference standard should preferably be a secondary standard, i.e., directly or indirectly traceable to a primary standard. If sending the instrument to NBS is impractical or uneconomical, it may be that sufficient accuracy for some quantity can be achieved locally without this process. This would still define a reference standard.

Working Standard

The next standard in the hierarchy is the working standard which is calibrated against the secondary or reference standard. It is used regularly to calibrate field instruments and instruments used in day-to-day operations. This standard has, of course, the largest imprecision among the calibration instruments, but it is more convenient to use and its calibration can be reestablished locally if instrument drifts are suspected, or if it is damaged.

Other types of standards are transfer standards which are used when transporting a secondary standard to NBS is impractical; traveling standards are specifically designed for convenient transportation in the field.

3.3 STANDARDS FOR FUNDAMENTAL METEOROLOGICAL VARIABLES

3.3.1 PRESSURE

The device, which NBS uses as their primary calibration standard, is the piston gauge. It can determine pressures from .01 to 10,000 psi with a resolution and calibration uncertainty in the parts per million range.

The piston gauge consists of a piston formed by a hollow cylinder through which fluid may enter and a weight carrier of minutely larger diameter fits over the piston. A small leakage of fluid between the weight carrier and the cylinder serves to reduce friction, and spinning the weight carrier further allows the system to slide freely. The entire assembly is used within an evacuated bell jar. As a result, fluid pressure is balanced (when the weight carrier is floating freely) by the weight of the carrier action over the interior diameter. The piston gauge thus yields pressure from the fundamental units of mass, length, and time.

Piston gauges are also used for secondary standards (e.g., NCAR). The sources of error to which the piston gauges are most sensitive are variations of local gravity and temperature (Cross, 1963). Effective gravity is a function of both latitude and altitude. Temperature is a consideration in the thermal expansion of the cylinder and the weight carrier areas. For usual materials, the area increases 13 to 18 ppm/°F (Dean, 1952).

Piston gauges are useful only as a calibration tool because the weight carrier is made to float by adjusting fluid pressure within the cylinder. Therefore, when using this device, one would adjust the pressure of a chamber to a fixed value and would not attempt to gauge an uncontrolled and continuously variable field, such as the atmosphere.

Mercurial barometers are commonly used as secondary or referenced standards. The calibration uncertainty of this device ranges from .001% to .03% of the reading, and is due to several sources. The first can be an error in reading the height of the mercury column, because a meniscus develops due to capillary effects which can be minimized by increasing the mercury column diameter. With careful lighting, heights can be read to .001 inches. The thermal expansion of mercury must also be considered for careful work. In addition to calibration, the

mercury barometer can also be used as an ambient measurement device, unlike the piston gauge.

Field or working standards are almost always high-quality aneroid barometers which can be trusted to about 0.25 mb.

See the American National Standards Institute's ANSI/ASTM D3631-77 Standard Methods for Measuring Surface Atmospheric Pressure.

3.3.2 TEMPERATURE

The concept of temperature is rooted in the axioms of thermodynamics (e.g., Benedict, 1977; Schooley, 1982). In order to define a thermodynamic temperature scale, one must either assign a temperature value (say, 273.15) to a fixed-reference point (freezing point of water), or assign a temperature difference (say, 100) to a fixed-reference point (boiling and freezing points of water) and then thermodynamically determine temperature ratios between other temperatures and these fixed-reference points. This is exceedingly difficult and cumbersome to do experimentally because there is no ideal gas and, as a result, practical temperature scales have been invented which define reference fixed points and specify procedures for interpolation between them.

The most recently-defined scale is the International Practical Temperature Scale of 1968 or IPTS-68 [Metrologia, 5, (1969) and 12 (1976)]. The defining fixed points are given in Table 3-1 (Benedict, 1977), along with the interpolating instrument for each temperature range. Unless otherwise indicated, fixed points are for pressures of one atmosphere.

NBS bases their primary temperature standard on these fixed points. For interpolation between the points in the range of interests to meteorology, NBS uses a platinum resistance thermometer (PRT) made of a coil of ultra-pure platinum annealed to relieve any strain. Above 0°C an equation referred to as the Callendar-Van Dusen relationship, modified by the addition of an error function, is used to relate temperature and resistance. Below zero a reference function defines the relationship. The uncertainty of the NBS standard is of the order of microkelvins.

Secondary or reference standards are generally high quality PRT's (e.g., NCAR) or liquid-in-glass (e.g., ASTM thermometers with calibration certificates). With high precision electronics, an accuracy of the order of millidegrees Celsius or millikelvins can be achieved for a laboratory PRT. Resolution generally prevents liquid-in-glass thermometers to reach this level of accuracy. For laboratory calibration of standards, special fixed-temperature generators are available such as the gallium cell (NBS Special Publication #260). Working standards are generally ventilated PRT or liquid-in-glass thermometers, accurate to about 1/10K.

Table 3-1. Primary Fixed Points of the IPTS-68

Defining fixed point	Temperature (K)	Interpolating Instrument
Triple point, hydrogen	13.81	
Boiling point, hydrogen (25/76 atmospheres)	17.042	
Boiling point, hydrogen	20.28	Platinum
Boiling point, neon	27.102	Resistance
Triple point, oxygen	54.361	Thermometer
Boiling point, oxygen	90.188	
Triple point, water	273.16	
Boiling point, water	373.15	Platinum
Freezing point, zinc	692.73	Resistance
Freezing point, silver	1235.08	Thermometer
Freezing point, gold	1337.58	Platinum/ 10% Rhodium- platinum
	>1337.58	Thermocouple
		Optical Pyrometer and Planck's Radiation Law

3.3.3 WIND SPEED

NBS maintains a low-speed and a high-speed wind tunnel for calibration of anemometers. The low-speed wind tunnel is designed for speeds between 0.06 and 9 mps with a cross section of 0.9 x 0.9 m. The high-speed wind tunnel has two sections one 1.5 x 2.1 m for speeds between 2 and 46 mps, the other 1.2 x 1.5 m for speeds between 4 and 82 mps. In the low-speed wind tunnel, a laser Doppler anemometer is used to determine the velocity, while the other has a pitot tube (Purtell and Klebanoff, 1979). Pitot tubes are generally used for secondary or reference standards in wind tunnels, while well-calibrated cup or propeller anemometers often serve as working standards. At lower speeds, where the response of the pitot tubes are inadequate, calibration of such instruments as hot-wire anemometers for indoor flow measurements can be done by timing soap bubbles, smoke puffs, or balloons, or mounting the anemometer on a disk which is rotated at a low known rate or moving the anemometer at a known rate in a room with still air.

Though definitely inferior to wind tunnel calibration of anemometers, field calibrations, using a constant rpm motor to drive

the anemometer, are commonly used by the U.S. Weather Service. While this method calibrates the generator or tachometer mechanism, changes in the dynamic response of the anemometer cups or bearing friction are ignored, which can easily produce significant changes in the anemometer response.

3.3.4 HUMIDITY

NBS has selected a gravimetric method for setting their primary standard because it measures humidity in fundamental units. In this method, the water vapor mixed with a gas is absorbed by a desiccant and the increase in weight is measured. The mixing ratio between the mass of the absorbed water vapor and the dry air volume-density product is then calculated with an accuracy of 0.1 percent. The density is determined from the measured temperature and pressure (Wexler, 1965). Dew cells are often used for secondary standards. However the complexity of properly operating this type of instrument has forced many institutes to use psychrometers as secondary or reference standards, at least for temperatures above freezing. With carefully calibrated thermometers, properly treated wicks, sufficient ventilation and minimization of radiation errors, relative humidity uncertainties of +2% absolute or less are achievable in the laboratory. Working standards are generally radiation shielded and fan aspirated; a sling psychrometer is less accurate.

See ASTM D4023-82a Standard Definitions of Terms Relating to HUMIDITY MEASUREMENTS; ASTM E337(83) Standard Method for Measuring Relative Humidity with a Psychrometer (Wet- and Dry-Bulb Hygrometer); ASTM E104(75) Standard Recommended Practice for Maintaining Constant Relative Humidity by Means of Aqueous Solutions.

3.3.5 RADIATION

Though NBS is involved in radiation standards work, they have left solar and atmospheric radiation to the meteorological community. WMO operates a primary standard, an absolute cavity pyrheliometer, in Davos, Switzerland. This instrument is claimed to have an accuracy of +0.2%. Secondary standards of the same type from the various WMO regions are calibrated against this standard every five years. These standards claim an accuracy of +0.5%. The WMO region four, to which the U.S. belongs, has its standard at the National Oceanic and Atmospheric Administration (NOAA) in Boulder. Radiation standards from universities and other institutions are regularly sent to this facility for side-by-side comparisons. Regression equations or daily total radiation ratios are developed for standards which are generally Eppley PSP pyranometers or Eppley NIP pyrheliometers.

3.4 OTHER STANDARDS LABORATORIES

3.4.1 NONCOMMERCIAL LABORATORIES

While NBS strives towards the ultimate accuracy and precision in measurements, other institutions and agencies create and maintain standards with accuracy or bias and precision defined for practical applications. Besides maintaining these standards (see the annual ASTM Book of Standards) through the work of volunteers, these groups also publish standards of a wide variety.

For meteorology, the most important of these groups is probably the ASTM (1916 Race Street, Philadelphia, PA 19103). Their committee D-22 on Sampling and Analysis of Atmospheres has a subcommittee called D-22.11 Meteorology. They are currently developing standard methods for:

- Atmospheric temperature measurements.
- Relative humidity measurements with a cooled surface condensation hygrometer.
- Performance of cup or propeller anemometers.
- Dynamic performance of a wind vane.
- Operational comparability of two meteorological systems.

These standards, when published, should prove very useful for defining accuracy or bias and precision for field work.

Other agencies that generate a concern or requirement for meteorological measurements of known quality are the Environmental Protection Agency (EPA OAQPS MD-14 and EMSL MD-56, Research Triangle Park, NC 27711), NOAA (NOAA/NWS, Test and Evaluation Laboratories, SR and DC RD No. 1, Sterling, VA 22170), NCAR (NCAR/Field Observing Facility, P.O. Box 3000, Boulder, CO 80307), and a number of societies and agencies involved with nuclear power generation safety. Besides developing requirements and making recommendations, these agencies also sponsor meetings and workshops. This is also done by societies such as the Precision Measurement Association. The World Meteorological Organization has published a number of reports on instrumentation and measurement standards mainly as Technical Notes (Secretariat of the WMO, Geneva, Switzerland).

3.4.2 COMMERCIAL LABORATORIES

Most instrument manufacturers maintain their own calibration facilities; however, calibration documentation in advertisements are often poor at best and erroneous at worst. It is unfortunate that no set of rules or recommendations for advertising instrument performance exist or are enforced.

The term "accuracy" is often abused by commercial instrument manufacturers. For example, if one advertises the "accuracy" of a temperature sensor as being ± 0.5 K, the tendency is to read the value as being the largest error one would ever encounter from the instrument. In general, such a specification is determined from the standard deviation of residuals from a least-squares fit of a calibration curve. As such, the ± 0.5 K represents the interval within 63% of the time, assuming normal distribution of deviation. It is better to specify precision (not accuracy) in terms of 2σ or 3σ intervals or percent confidence intervals about a calibration curve. The size of the interval, without specification of what sort of interval it is, is meaningless. In addition, a complete description of an instrument's performance also requires a specification of its long-term stability. This quantity is usually presented in meteorological units per year. In general, long-term stability is influenced more by physical changes in an instrument, such as bearing wear in cup anemometers, window aging in Lyman-alpha hygrometers, etc., than by random changes and will therefore appear as a more or less monotonic drift--barring rough handling of the instrument. It is however often difficult and time consuming to get the data required for long-term stability analysis.

Often only the prototype or a few samples of a series of instruments may have been tested to arrive at the advertised "accuracy." Individual instruments may be significantly less precise or accurate. Another question is how the manufacturer determines the "true" value. Often it claims that its calibrations are "NBS traceable." NBS defines this concept as "the ability to relate individual measurement results to national standards or nationally accepted measurement systems through an unbroken chain of comparison" (NBS Special Publication No. 250). The concept of traceability is however often misused as, for example, a long chain of undocumented calibrations with compounding uncertainties does not warrant the acceptance of "accuracy" as the phrase implies. It should also be noted that NBS in no way guarantees the accuracy or precision of a commercial instrument, even if it is "NBS traceable."

3.5 QUESTIONS AND PROBLEMS

Three clocks were tested by comparing them with a very accurate clock. Table 3-2 shows the number of seconds each clock was fast for each week of the test. (Clocks 4-6 are included for optional use.)

1. Plot these data. Describe the various types of inaccuracies shown by each of the clocks and estimate the magnitude of each.

2. Determine calibrations for the clocks. (Classes with sufficient statistics should find confidence intervals for their calibrations.)
3. Rank the clocks from best to worst when used with the calibration and discuss your rankings. How would the rankings change if the clocks were to be used without the calibration? Discuss how the rankings depend on the use made of the clocks and list uses of which each of the clocks would be most or least suitable.

Table 3-2. Comparison of Three Clocks

F. Clock: Program to generate data for clock problem

Week	Clock					
	1	2	3	4	5	6
0	0.00	0.00	0.00	0.00	0.00	0.00
1	2.57	-0.82	-125.52	-0.91	-0.02	0.64
2	12.30	25.47	-118.10	-1.14	2.05	0.52
3	-4.09	49.28	-133.60	-0.89	2.60	0.13
4	7.34	74.89	-128.59	0.16	2.75	-0.31
5	9.83	101.07	-128.62	0.64	4.92	-0.53
6	-7.10	124.12	-133.24	0.21	5.69	-0.51
7	22.66	150.05	-139.10	1.30	7.16	-0.41
8	-5.08	175.16	-148.58	-0.27	8.78	-0.88
9	-4.13	199.18	-143.87	-0.63	10.30	-1.03
	NWeek = 10		SFED = 4.00000			

Answers:

These data were created using $Y_i = \text{GAIN} * \text{WEEK} + \text{BIAS} + \text{RANDOM ERROR}$ where RANDOM ERROR was a random number with standard deviation STAND ERROR.

Calibrations and confidence intervals for the calibration gain and bias are shown in Table 3-3.

BIASN is the offset of the clocks at the last week. This is an important parameter because it could be used to reset the clocks. The confidence interval for BIASN thus tells how well the offset of the clocks could be set based on this calibration.

The large amount of random noise in clock 1 not only produces a large random error but also gives a large uncertainty in the gain. Both of these are sources of errors. After several weeks, the uncertainty in the gain can cause a considerable error.

When used with its calibration, clock 2 is most precise, but its rate of gain will cause problems if it must be used without a calibration and if frequent resetting of the bias is not possible. If the gain rates of the clocks can be adjusted, this problem can be overcome.

If clocks 4, 5, and 6 are used, note that the magnitudes of the gains of clocks 4 and 6 are not significantly different at the 90% level.

Table 3-3. Calibration and Confidence Intervals

CALIBRATION OF CLOCKS OBTAINED BY LEAST-SQUARES FIT TO THE DATA

Clock	1	2	3	4	5	6
GAIN (sec/week)	-0.25	25.04	-3.04	-0.01	1.23	-0.22
90.000% Conf (+)	2.06	0.15	0.95	0.21	0.10	0.04
BIASO (sec)	5.36	-0.28	-121.12	-0.33	0.01	0.59
BIASN (sec)	3.13	225.08	-148.51	0.41	11.06	-1.38
90.000% Conf (+)	6.75	0.48	3.10	0.70	0.34	0.13
STAND ERROR (sec)	9.50	0.68	4.37	0.99	0.48	0.18

GAIN = RATE CLOCK GAINS TIME

BIASO= OFFSET OF CLOCK AT WEEK 0

BIASN= OFFSET OF CLOCK AT WEEK N WEEK

STAND ERROR IS STANDARD ERROR OR FIT FOR CALIBRATION

90.000 CONFIDENCE INTERVALS ARE FORMED BY USING

GAIN + CONF, BIASO + CONF, AND BIASN + CONF

CONF FOR BIASO AND BIASN ARE EQUAL

DATA WERE GENERATED USING RANDOM ERRORS AND THE FOLLOWING PARAMETER VALUES:

Clock	1	2	3	4	5	6
GAIN (sec/week)	0.00	25.00	-3.00	0.00	1.23	-0.21
BIASO (sec)	0.00	0.00	-120.00	0.00	0.00	0.57
BIASN (sec)	0.00	225.00	-147.00	0.00	11.07	-1.32
STAND ERROR (sec)	20.00	0.50	4.00	1.00	0.49	0.20

Historical Note

Before radio navigation, accurate ships' clocks were needed for navigation. Each second error in the ship's clock would produce nearly a 0.5 km error in the ship's position as determined by the sextant for a ship near the equator. Considerable effort was made to develop chronometers which would keep accurate time aboard a rolling ship. Captains had their clocks compared with observatory clocks and kept careful records. In the book Mutiny on the Bounty, Captain Bligh sent first mate Fletcher Christian to have the Bounty's clock compared with the clock at the Greenwich Observatory. Christian again compared this clock with the Capetown Observatory.

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CHAPTER 4

MEASURING DEVICES

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4.0 INTRODUCTION

The performance characteristics of individual meteorological sensors are presented in this chapter. Although emphasis will be given to commonly used sensors, devices that have historical or specialized significance will be identified. The list of this latter group may not be exhaustive.

A basic understanding of instrument characteristics requires fundamental knowledge of college-level physics. For most sensors, concepts from calculus will be used, and, for some, an understanding of ordinary linear differential equations is assumed. Elementary vector analysis is needed for resolution of vectors into components and for relating forces and torques.

Some laboratories will assume the student is familiar with BASIC or FORTRAN. For others, access to and familiarity with a programmable calculator (or equivalent) will be assumed.

Each subchapter focuses on one meteorological variable and has five sections that describe the variable and the fundamentals of sensors used to assess its characteristics.

Section 1 gives the required concepts the student is expected to have mastered in previous courses and which will be used to describe the characteristics of the meteorological variable and the devices used to measure it. Items will be listed and referenced but will not be discussed in detail.

Section 2 gives the theoretical background for understanding the static and dynamic behavior of the variable and its measurement. Although complete derivations may not in every case be provided, sufficient detail and/or references will be given to provide a basic framework for lectures to precede the instrumentation laboratory.

Section 3 provides descriptions and calibration procedures for individual sensors or transducers. Devices most commonly used in practice will be given more emphasis than specialized, obsolete, or impractical instruments.

Section 4 covers material on specifications and selection of sensors to match observational constraints and objectives. Unique siting considerations, sampling, data processing, and recording strategies will be discussed.

Section 5 provides a variety of questions that could be used as discussion questions in class or as assignments or as test questions. In addition, it gives a list of problems that may be assigned to the students and a compilation of laboratory exercises and in-class demonstrations, which is being made available to course instructors for adoption, modification, or use as they see fit.

References to current and historical literature relating to the specific sensors and their use in meteorological measurements are listed at the end of the chapter.

4.1 TEMPERATURE

4.1.1 INTRODUCTION

This section covers the measurement of air temperature with emphasis on the types of sensors commonly used for in-situ measurement near the surface of the earth. The sensors are described by the physical principle employed starting with thermal expansion types and includes thermoelectric types and resistance thermometers. There is relevant material in earlier sections. The dynamic response of temperature sensors was described in Section 2.2, while calibration and standards were discussed in Chapter 3.

For convenience, some useful temperature relations and calibration points, taken from Fritschen and Gay (1979), are listed in Table 4-1.

Table 4-1. Several near-ambient temperature references with various temperature scales. Pressures are 1013.25 hPa for the ice and steam points and 61.06 hPa for the triple point.

<u>Scale</u>	<u>Symbol</u>	<u>Ice Point</u>	<u>Triple Point</u>	<u>Steam Point</u>
Thermodynamic	K	273.15	273.16	373.15
Celsius	C	0.00	0.01	100.00
Fahrenheit	F	32.00	32.018	212.00

4.1.2 REQUIRED BACKGROUND

This section requires mathematics at least through calculus and some differential equations. The instructor can delete some material if desired to eliminate the requirement for differential equations. One year of physics and some meteorology are also required. Temperature measurements are made in many fields and consequently the sensors and measurement techniques are highly developed. The major problem for measurement of air temperature is the poor coupling with the atmosphere. Knowledge of the properties of the atmosphere and the radiation environment is required to make good air temperature measurements.

4.1.3 TEMPERATURE SENSORS

Thermal-Expansion Methods

Bimetallic thermometer. Two strips of different metals with different thermal-expansion coefficients are bonded together while at the same temperature. A subsequent temperature change causes differential expansion and the strip will deflect in a uniform circular arc. Analysis of a bimetallic strip requires use of experimentally determined factors. The details of materials and bonding processes are usually trade secrets. These sensors are not very precise but are useful as temperature controllers (thermostats), indicators, and inexpensive recorders. A high quality device may have an inaccuracy of about 1% of scale range.

Liquid-in-glass thermometer. Liquid-in-glass thermometers are classified according to the immersion required and the liquid used. The immersion classes are as follows:

- Partial Immersion. Bulb and small indicated portion of the stem is immersed.
- Total Immersion. Bulb and the portion of the stem containing liquid is immersed.
- Complete Immersion. Entire thermometer is immersed (bulb and all of the stem).

A complete immersion thermometer must be used for measurement in the atmosphere. Partial immersion thermometers are often used for laboratory calibrations. This type of thermometer makes use of differential expansion of a liquid, such as mercury or alcohol, with respect to glass. Mercury is used only above -38°F , while alcohol can be used down to -80°F .

Maximum thermometer is usually mercury-in-glass with a constriction in the stem. An increase in temperature forces the mercury past the constriction, but when the temperature falls the column breaks, thus indicating the maximum temperature since last reset. It must be mounted with the bulb slightly higher than the rest of the column, because the mercury column will otherwise tend to retract slightly before breaking as the temperature drops.

Minimum thermometer is typically alcohol-in-glass with a small dark dumbbell immersed in the liquid in the bore. Surface tension keeps the index in the liquid, thus pulling the index toward the bulb as the temperature drops. It must be mounted in a horizontal position. When the temperature increases, the liquid flows around the index, which does not completely block the bore. It is held in place by friction.

Best possible accuracy for a liquid-in-glass thermometer is with mercury, and error can be as low as 0.002°C .

Pressure thermometer. These thermometers have a sensitive bulb containing mercury or xylene connected by a capillary tube to a pressure measuring device. The tube may be up to 200 ft long. Pressure is measured by a Bourdon tube, bellows, or diaphragm.

Thermoelectric Devices (Thermocouples)

The advantages of thermocouples include wide useful temperature range, fast response, ruggedness, reliability, and low cost. However, they exhibit rather poor accuracy. If all thermocouples used in a system are constructed from the same spool of premium wire, then the same calibration should apply to all. They are low impedance devices and are readily interfaced to recorders for a simple measuring system. Their main disadvantages are very low output, on the order of $40 \mu\text{V}/^{\circ}\text{C}$, a slight nonlinearity, and the need for empirical calibration.

The simple thermocouple circuit shown in Fig. 4-1 comprises two junctions formed by the union of dissimilar metals. The thermal electromotive force (emf) observed at the terminals as a result of the temperature difference T_1-T_2 is called the "Seebeck effect."

The above emf will appear if no current is allowed to flow through the circuit. If a current does flow through the circuit, then one junction will be heated and the other will be cooled. This is called the "Peltier effect." In addition, as with any other circuit, current flow will release heat proportional to I^2R . The current effects can be made negligible by reading the emf with a high impedance voltmeter.

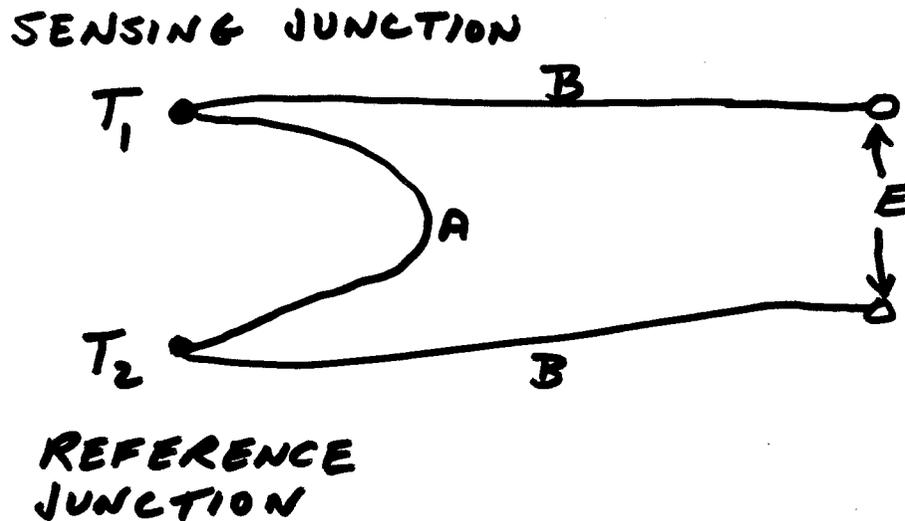


Fig. 4-1. Thermocouple circuit.

Thermocouple Behavior Laws:

1. The thermal emf of a thermocouple is unaffected by temperatures elsewhere in the circuit, if the two metals used are each homogeneous. Thus one can use lead wires made of thermocouple metals.

2. If a third metal is inserted in either wire A or B (see Fig. 4-1) and if the two new junctions are at the same temperature, no effective emf is generated. This means that we can measure the emf with a real voltmeter since the meter terminals are usually copper and close together and are at the same temperature.

3. If a metal C is inserted in one of the AB junctions, then no net emf is generated so long as junction AC and BC are at same temperature. Therefore, a junction can be soldered. The solder wicks between twisted wires form the AC and BC junctions, where an AB junction was intended.

4. If the thermal emf of metals A and C is E_{ac} , and junctions B and C are E_{bc} , then for metals A and B it is $E_{ac} + E_{bc}$. In order to calibrate all desired metal pairs, it is only necessary to calibrate all of the desired metals against a standard.

5. If a thermocouple produces emf E_1 when the junction temperatures are at T_1 and T_2 , and E_2 when at T_2 and T_3 , it will produce $E_1 + E_2$ for junction temperatures T_1 and T_3 . Standard tables or equations can be used, even if the reference junction is not at the ice point.

Two types of temperature measurement are possible with thermocouples: differential and absolute. Thermocouples inherently measure the differential temperature between two junctions. Absolute temperature measures one junction, known as the reference junction, which must be held at a known temperature. There are several ways of accomplishing this.

1. Maintain the reference junction at the triple point of water. There are commercially available cells which maintain the triple point, $0.01^\circ\text{C} \pm 0.001^\circ\text{C}$.

2. Maintain the reference junction at the ice point. An ice bath consisting of a mixture of melting, shaved ice and water, can be constructed to bring the reference junction to $0.00^\circ\text{C} \pm 0.0001^\circ\text{C}$ with extreme care. But $\pm 0.01^\circ\text{C}$ is more usual and $\pm 1.0^\circ\text{C}$ may be the result of less care.

3. Put the junction in an oven at a known temperature then add a constant to the resulting measured emf. According to thermocouple behavior law number five, this is equivalent to keeping the junction at 0°C .

4. Leave junction at ambient temperature, measure junction temperature, and add a compensating factor as above.

The commonly used thermocouple types are identified by letter designations originally assigned by the Instrument Society of America and adopted as an American Standard in ANSI MC 96.1, see the ASTM Manual of the Use of Thermocouples in Temperature Measurement (1981).

- Type T - Copper vs constantan
- Type J - Iron vs constantan
- Type E - Nickel, 10% chromium vs constantan
- Type K - Nickel, 10% chromium vs nickel, 5% aluminum
and silicon
- Type R - Platinum, 13% rhodium vs platinum
- Type S - Platinum, 10% rhodium vs platinum
- Type B - Platinum, 30% rhodium vs platinum, 6% rhodium

Frequently used in meteorological applications, type T thermocouples are resistant to corrosion in moist atmospheres. Since one wire is copper, there are fewer problems with extension wires and connections to voltmeters. The limits of error for Type T standard thermocouples are $+ 0.8^{\circ}\text{C}$ for standard wire in the meteorological temperature range and $\pm 0.4^{\circ}\text{C}$ for special wire.

A useful model for a type T, copper-constantan thermocouple, is the following

$$E = (a + bT)T \quad (1)$$

where T = temperature in $^{\circ}\text{C}$, E is the open circuit thermal voltage in μV , and $a = 38.63043$, $b = 0.04132299$. Some values are listed in Table 4-2.

Table 4-2. Thermocouple voltage in microvolts as a function of junction temperature in degrees C, assuming the reference junction is held at 0°C .

<u>Temperature</u>	<u>Voltage (μV)</u>
-30	-1121
-20	-757
-10	-383
0	0
10	391
20	789
30	1196
40	1611
50	2035

Figure 4-1 shows a typical thermocouple, and Fig. 4-2 is a plot of the data from Eq. 1. The slight nonlinearity of the thermocouple can be seen in this plot.

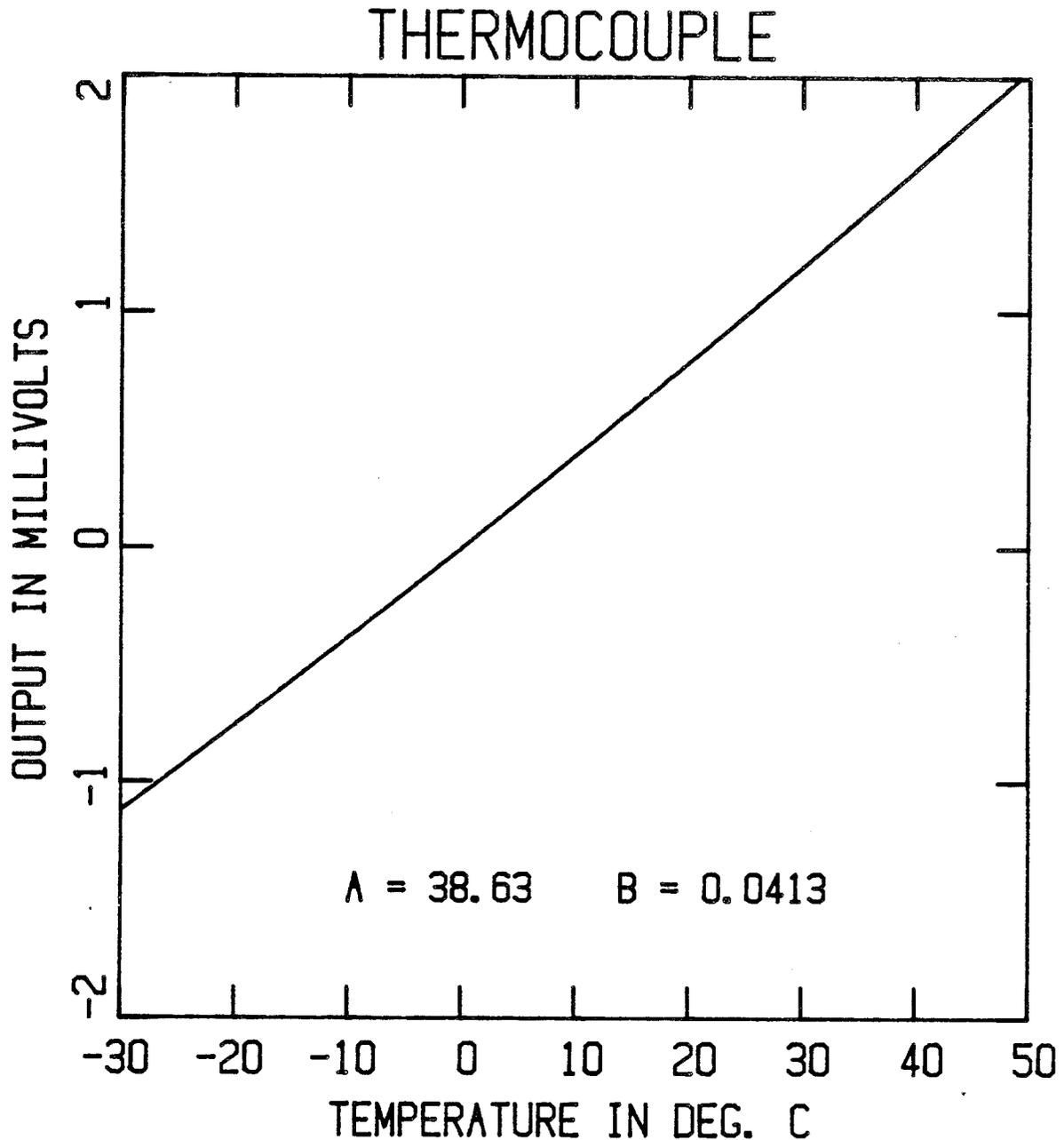


Fig. 4-2. Voltage generated by a copper-constantan thermocouple with the reference junction held at 0°C and with no load on the circuit.

Electrical-Resistance Sensors

Conductive devices (resistance thermometer). A resistance thermometer is a transducer whose resistance is a function of temperature. Platinum is used for precision resistance thermometers because it is stable, resists corrosion, is easily workable, has a high melting point, and can be obtained to a high degree of purity. It has

a simple and stable resistance-temperature relationship. Platinum is extremely sensitive to minute contaminating impurities and to strain. Platinum sensors are relatively high cost and typically have slow time response because of large size. There is a slight nonlinearity. They are low impedance sensors so it is often necessary to compensate for lead wires unless they are short and of constant length.

The resistance of a platinum temperature sensor is given by

$$R_T = R_0 (1 + aT + bT^2 + c(T - 100)T^3) \quad (2)$$

where R_0 = resistance in Ω at 0°C ,
 T = temperature of the sensor in $^\circ\text{C}$,
 R_T = resistance of the sensor at temperature T .

Typical values of the coefficients are

$$\begin{aligned} R_0 &= 100 \Omega, \\ a &= 3.90802 \text{ E-3}, \\ b &= -5.80195 \text{ E-7}, \\ c &= 0 \quad \text{for } T \geq 0^\circ\text{C}, \\ &= -4.27350 \text{ E-12} \quad \text{for } T < 0^\circ\text{C}. \end{aligned}$$

For the meteorological temperature range, here taken to be -30°C to 50°C , and usual precision, we can ignore the c term. Equation 2 with the above coefficients is plotted in Fig. 4-3, and the values are listed in Table 4-3.

Table 4-3. Temperature and resistance values for a platinum resistance sensor.

<u>Temperature ($^\circ\text{C}$)</u>	<u>Resistance (Ω)</u>
-30	88.22
-20	92.16
-10	96.09
0	100.00
10	103.90
20	107.79
30	111.67
40	115.54
50	119.40

The static sensitivity of this sensor can be found by taking the derivative of Eq. 1 with respect to temperature, which is equivalent to measuring the slope of the curve in Fig. 4-3.

$$\begin{aligned} \frac{dR_T}{dT} &= R_0(a + 2bT) & (3) \\ &= 0.39802 - 1.16039E-4 T \end{aligned}$$

which shows that the static sensitivity, or slope of the curve, changes very little with temperature over the range -30 to 50°C.

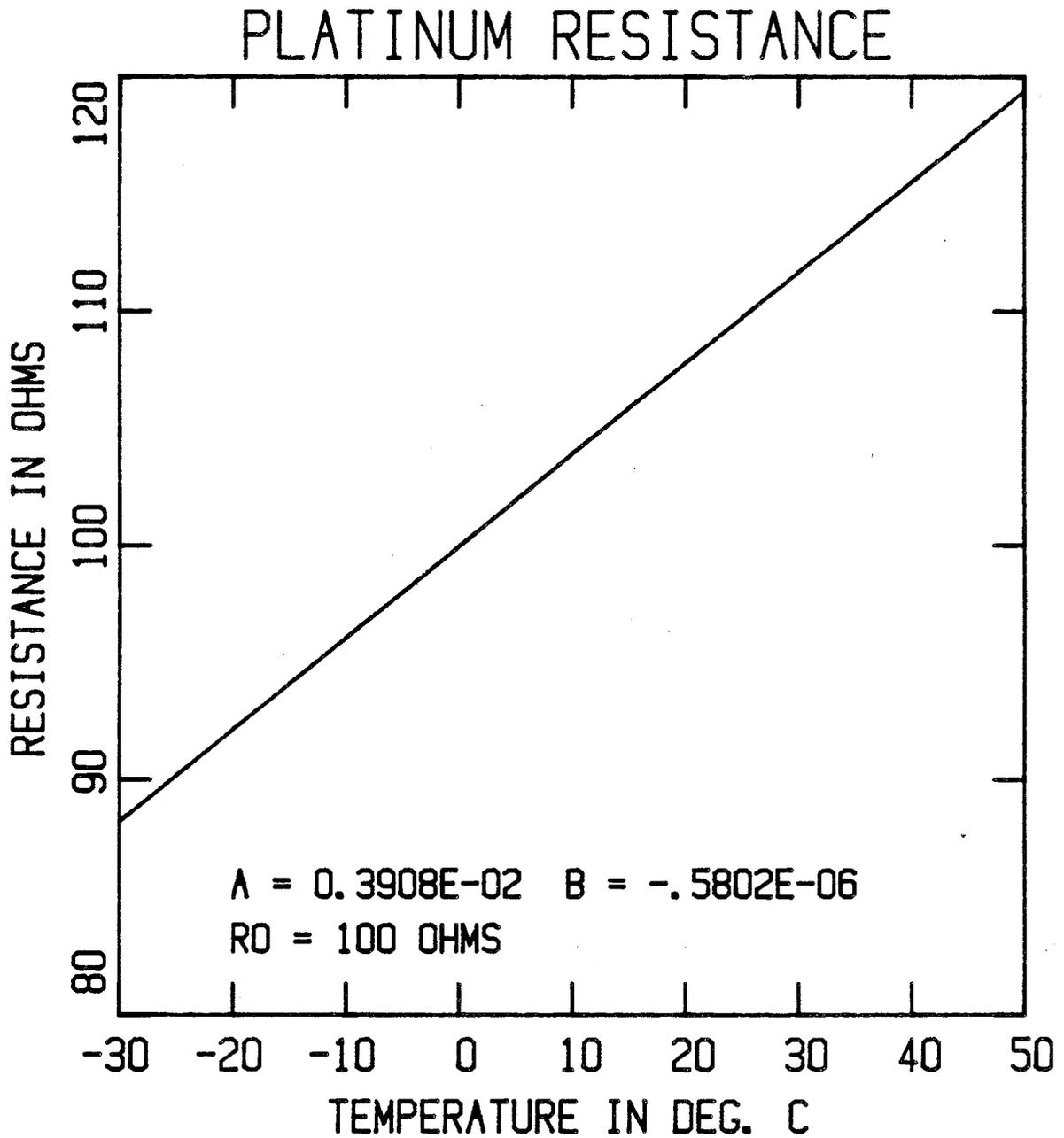


Fig. 4-3. Plot of typical platinum resistance temperature sensor resistance vs temperature.

Since the resistance of the sensor is low and the change with respect to temperature is small, it is common to use a platinum resistance thermometer (PRT) sensor in a bridge circuit such as the one shown in Fig. 4-4. The equation for this circuit is

$$V_0 = V_I G \left(\frac{R_T}{R_T + R_4} - \frac{R_2}{R_2 + R_3} \right) \quad (4)$$

where R_T is the PRT, and R_2, R_3, R_4 are fixed resistors, V_I is the input fixed reference voltage, G is the gain of the instrumentation amplifier, and V_0 is the output voltage. If we let $R_T = R(1 + x)$ where $x < 1$, and $R_2 = R_3 = R_4 = R$, then

$$V_0 = \frac{V_I G}{2} \left(\frac{x}{2 + x} \right). \quad (5)$$

From Eq. 2, we can see that $x = T(a + bT)$ and from Table 4-3 it is clear that $|x| < 0.2$. The combination of a PRT and bridge is shown in Fig. 4-5 where it is assumed that $V_I = 1.0$ V and that the instrumentation amplifier gain is 100. There is a noticeable curvature in this plot; evidently the nonlinearity of the bridge is much greater than that of the PRT itself. This nonlinearity must be corrected. It can be done with special analog circuits before recording the data, or if the data are to be digitized and processed in a computer, the nonlinearity can be corrected by a simple numerical algorithm.

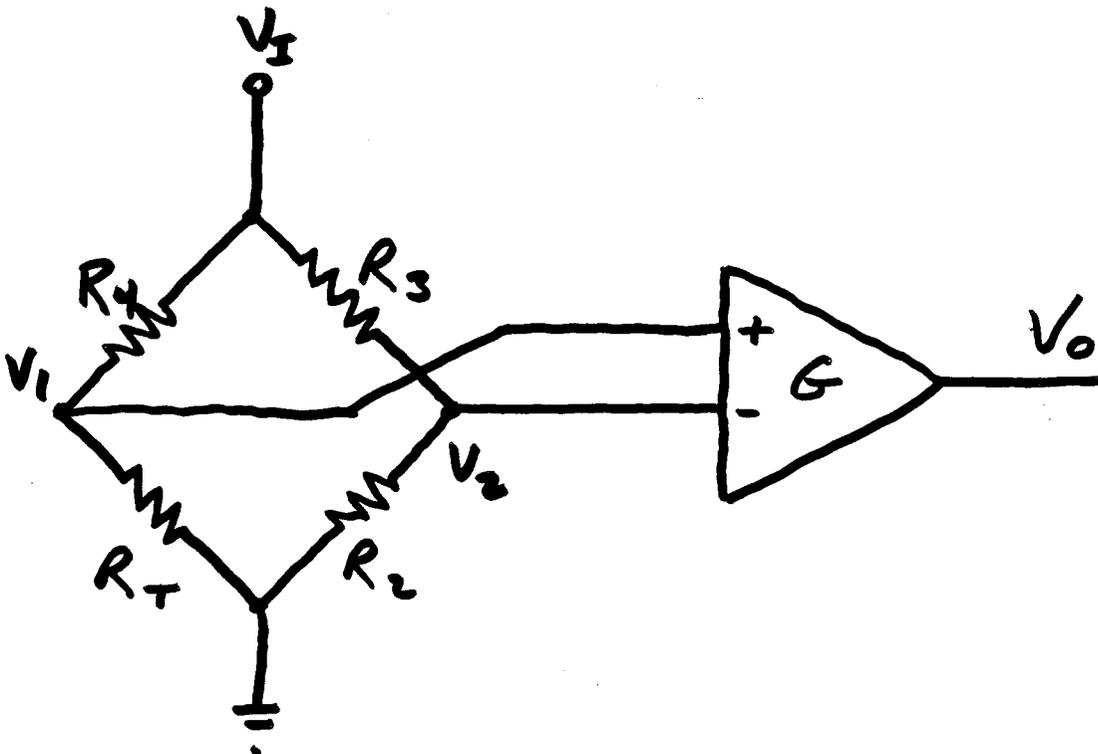


Fig. 4-4. Platinum resistance temperature sensor in a bridge circuit with an instrumentation amplifier.

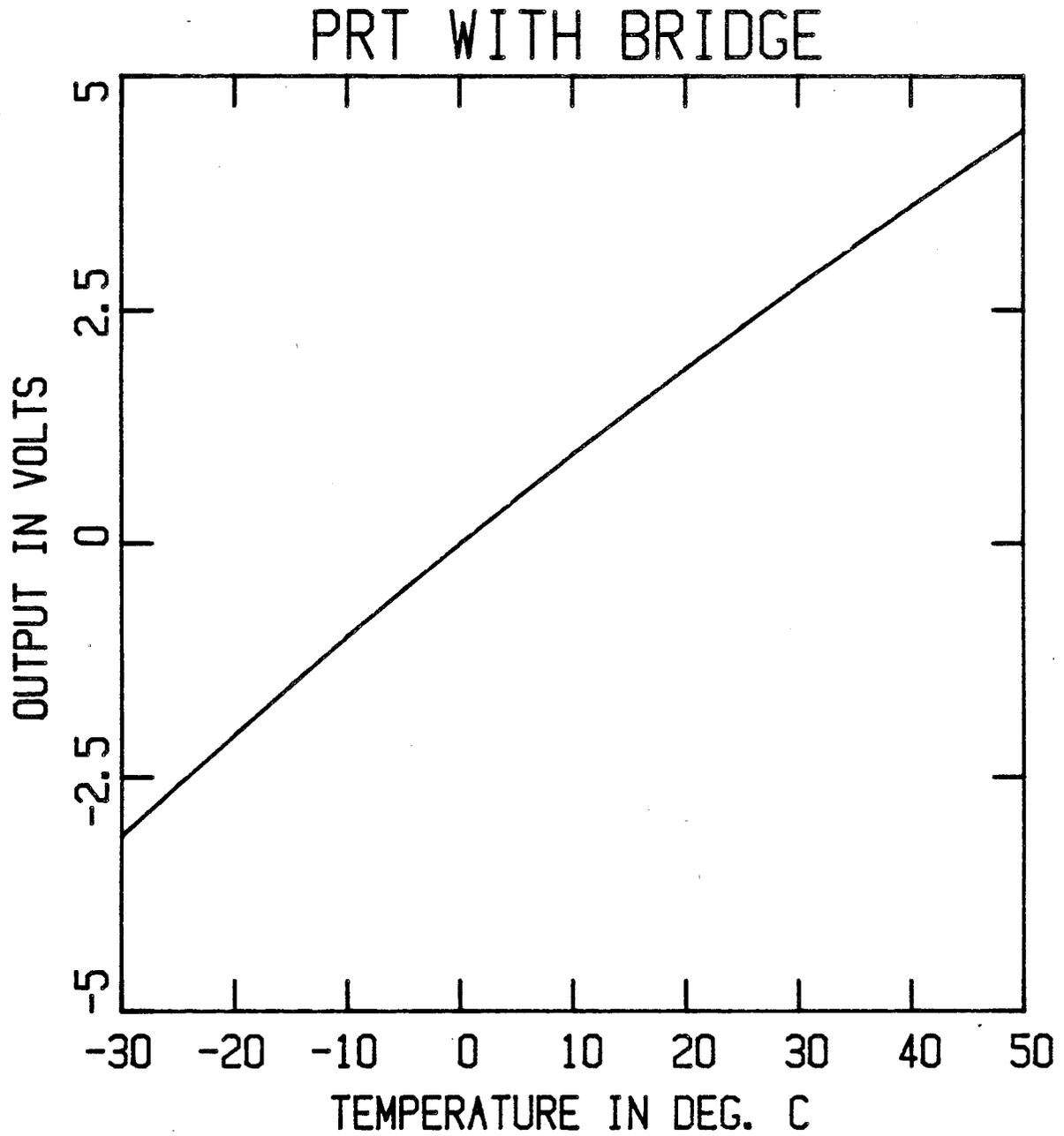


Fig. 4-5. Plot of the output signal of a platinum resistance temperature sensor in a bridge circuit with an instrumentation amplifier.

If long lead wires are used from the bridge to the sensor, the resistance and the resistance change with temperature of the copper wire may become significant. Three-wire and four-wire bridge designs are used to compensate for this. Higher resistance PRTs are available, up to 500 Ω , to alleviate this problem.

If too much current is allowed to flow through the sensor, this current may heat the sensor due to I^2R losses, which cause heating. It is important to keep this current low, typically less than 1 milliamp.

Note that the output signal in Eq. 4 is a function of the value of three fixed resistors, the instrumentation amplifier gain, and a fixed reference voltage. The latter two can change due to residual temperature sensitivity, change of components, supply voltage changes, or inadvertent adjustment. The PRT itself is very stable as long as it is kept dry, but the circuit components shown, and others that may be present, may not be nearly as stable especially in the field environment. This is, or should be, a matter of great concern, and it is possible to design the temperature measuring system to minimize these effects. For example, see Pike, Brock, and Semmer (1983).

Semiconductor devices (thermistor). Thermistors are temperature sensitive materials such as metallic oxides. They are semiconductors. They can be very stable. Thermistors have a large, nonlinear, negative coefficient of resistivity.

Several models for thermistors have been used; the following gives a very good fit over the range of temperatures used here.

$$1/T = a + b \ln R + c(\ln R)^3 \quad (6)$$

where T = temperature in $^{\circ}\text{K}$, R is the thermistor resistance and \ln is the logarithm to the base e . Consider two thermistors. For #1 the approximate resistance at 25°C is 6000 Ω , while for #2 the approximate resistance at 25°C is 30000 Ω . The coefficients for each are

$$\begin{array}{ll} \#1: & a = 1.240953 \text{ E-3} \\ & b = 2.360717 \text{ E-4} \\ & c = 8.998966 \text{ E-8} \end{array}$$

$$\begin{array}{ll} \#2: & a = 9.302871 \text{ E-4} \\ & b = 2.218025 \text{ E-4} \\ & c = 1.251305 \text{ E-7} \end{array}$$

and the resistance values as a function of temperature are shown in Table 4-4.

Table 4-4. Resistance as a function of temperature for two thermistors.

Temperature (°C)	#1	#2
-30	106200	481000
-20	58260	271200
-10	33200	158000
0	19590	94980
10	11940	58750
20	7496	37300
30	4834	24270
40	3196	16150
50	2162	10970

Figure 4-6 is a plot of the resistance-temperature relation for thermistor #1. Figure 4-7 shows a circuit designed to linearize the thermistor by using two thermistors. For example, thermistor #1 above could be R_3 and thermistor #2 could be R_4 . If $R_1 = 18700 \Omega$ and $R_2 = 35250 \Omega$, the result is shown in Fig. 4-8. The equation for the circuit in Fig. 4-7 is

$$\frac{E_2}{E_1} = \frac{R_4 (R_2 + R_3)}{R_4(R_1 + R_2 + R_3) + R_1(R_2 + R_3)} \quad (7)$$

and, to a first approximation,

$$\frac{E}{R} = \frac{E_2}{E_1} = 0.65107 - 0.0067966 T \quad (8)$$

where T = temperature in °C, and E_R is the voltage ratio. Figure 4-9 shows the deviation from the above linear approximation. The sensitivity of this circuit, the so-called linear thermistor, can be obtained from Eq. 8 as it is the derivative of E_R with respect to T which is $-0.0067966/^\circ\text{C}$.

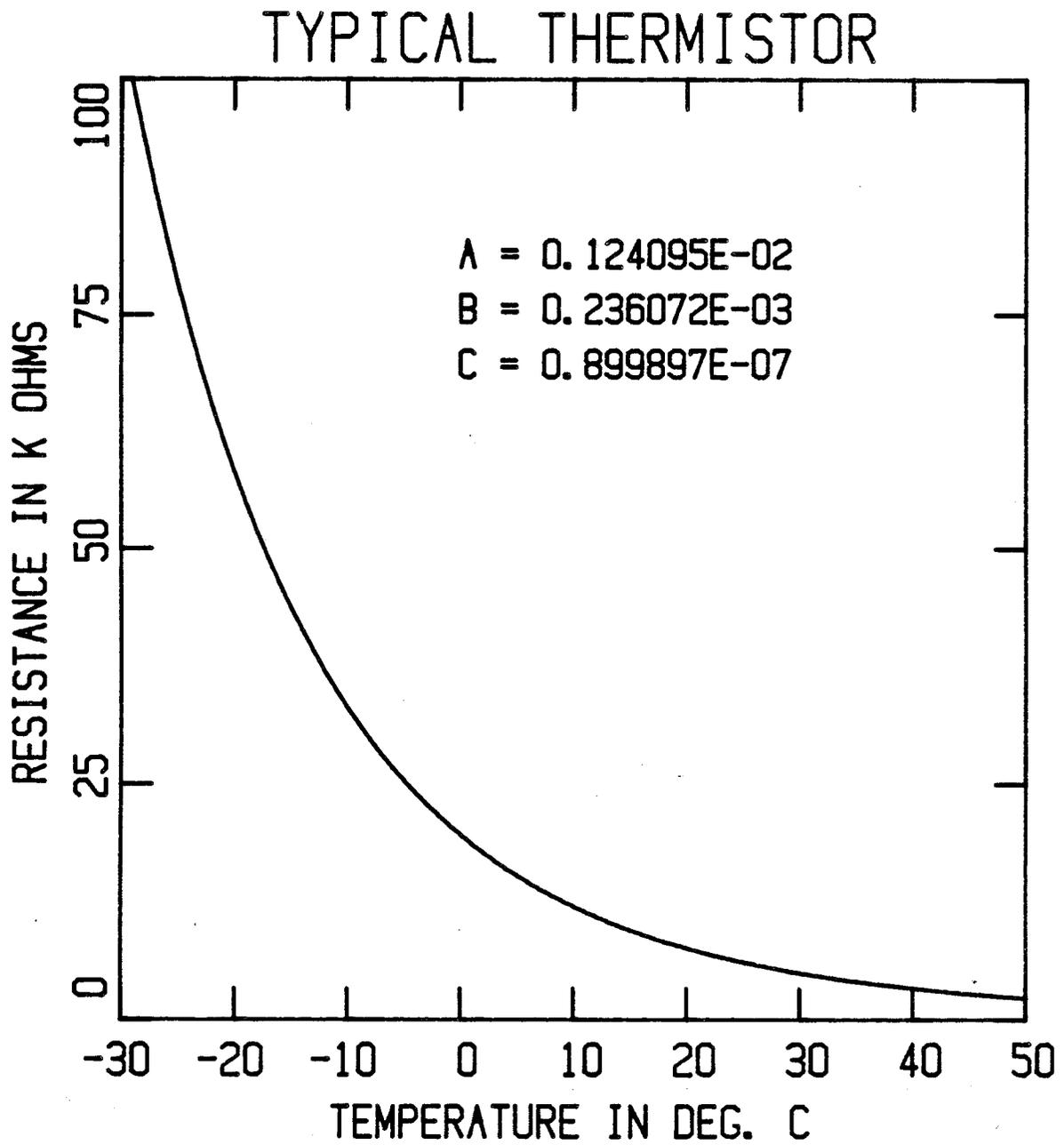


Fig. 4-6. Typical thermistor resistance vs temperature.

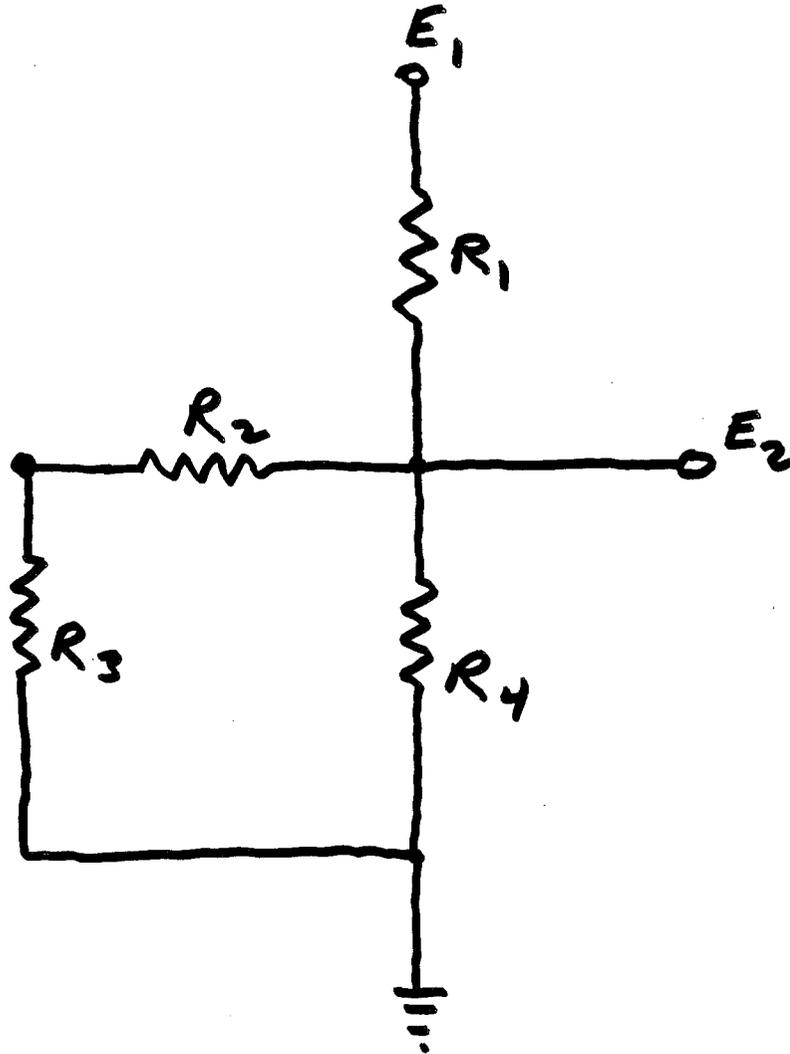


Fig. 4-7. Linear thermistor composite comprising two thermistors, R_3 and R_4 , in one sealed probe plus two external resistors R_1 and R_2 .

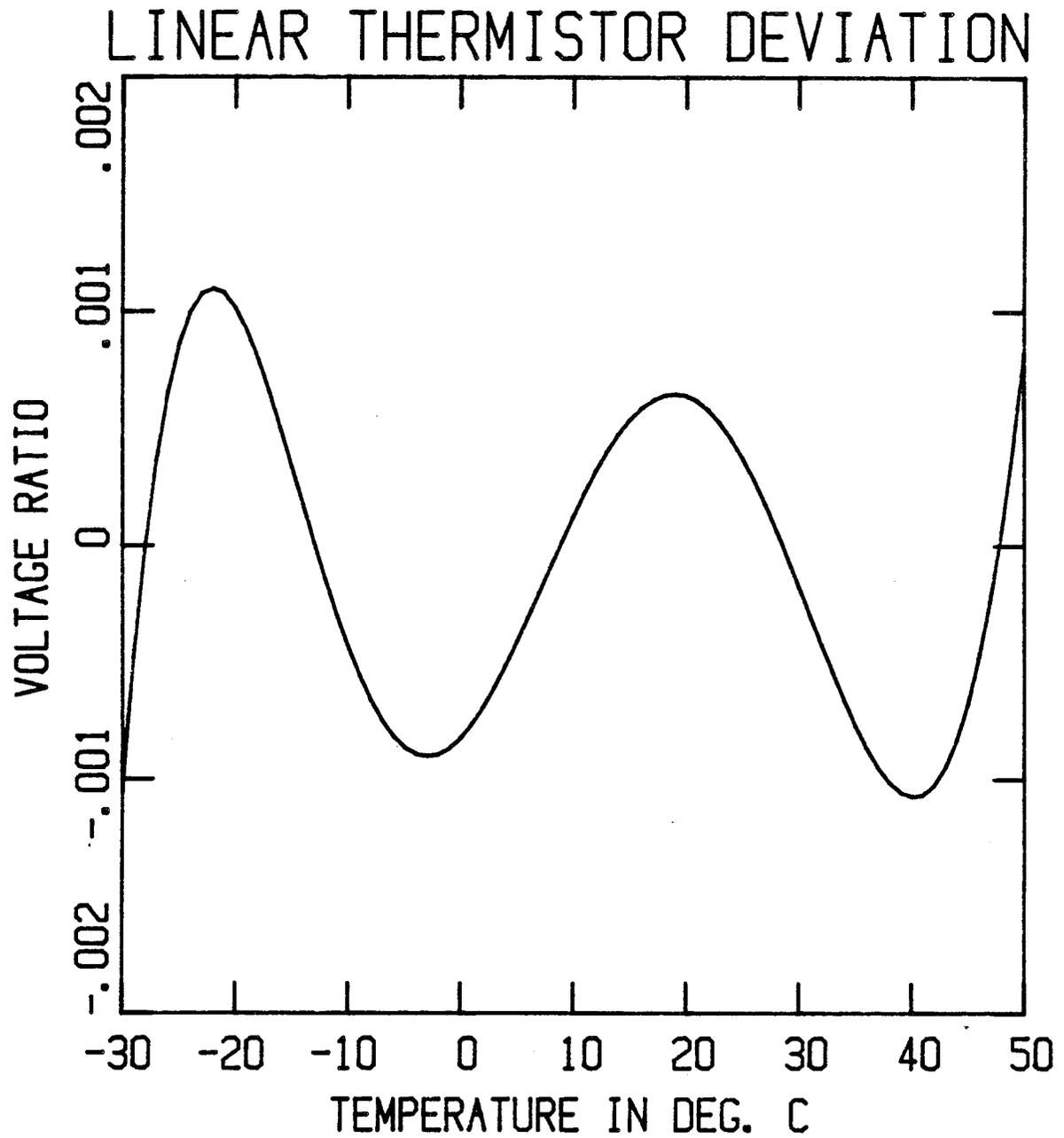


Fig. 4-8. Linear thermistor voltage ratio vs temperature.

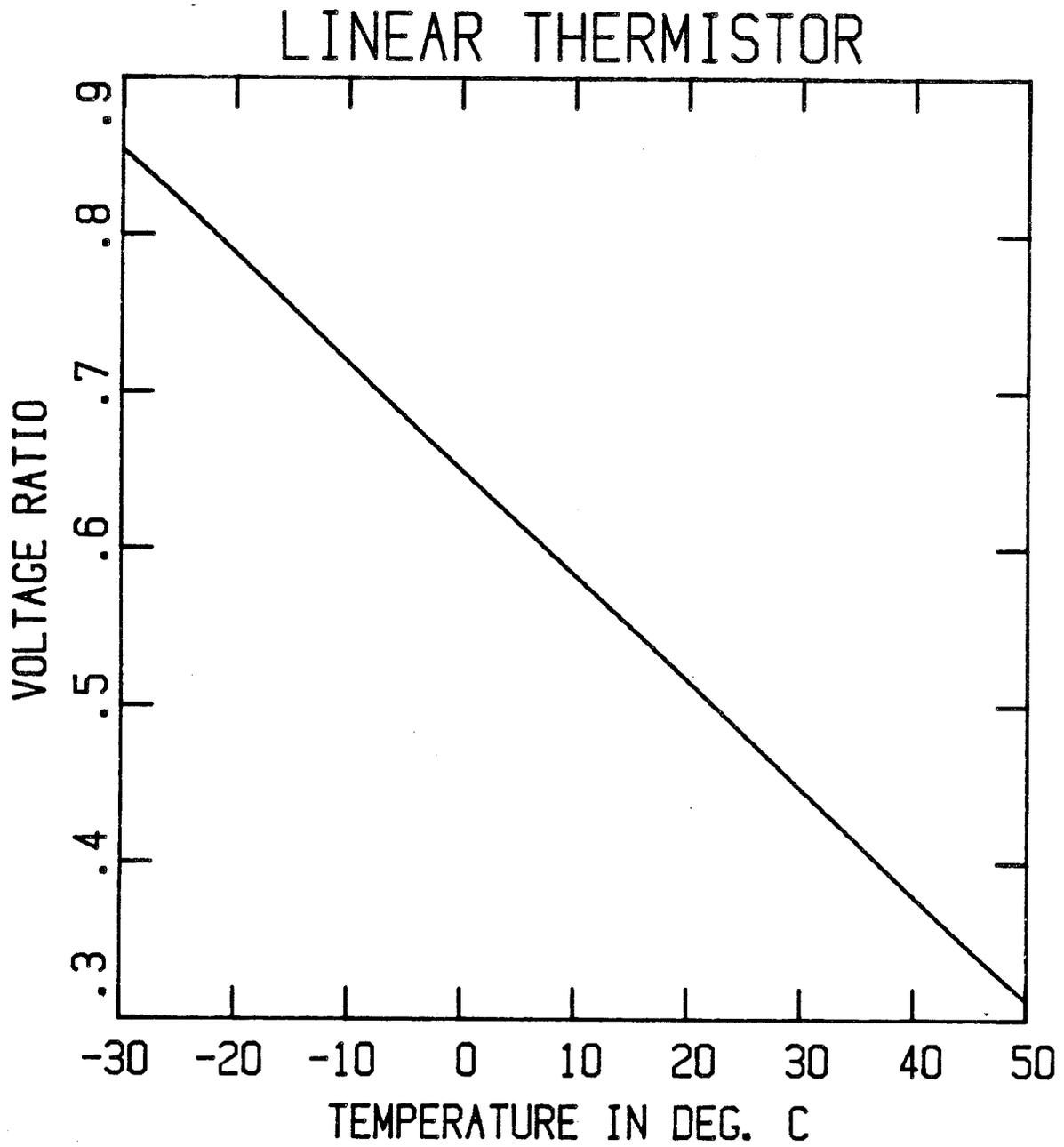


Fig. 4-9. Residual nonlinearity in the voltage ratio of a linear thermistor.

4.1.4 TEMPERATURE MEASURING PROBLEMS IN AIR

In air temperature measurement, the main problem is coupling with the atmosphere because air is a poor heat conductor. It is difficult to maximize convection, which brings the sensor to air temperature, while minimizing conduction along supporting hardware and radiation exchange with the sun, sky, and ground.

For ground based, in-situ instrumentation there will be no error because of aerodynamic heating. If the wind speed is too low, there will be an error because of failure to offset the conduction and radiation errors. Some temperature sensors are mounted in nonaspirated shields; this is called natural ventilation. Even the best nonaspirated shields cause significant errors in light wind conditions especially on clear, calm days and nights. The very best shields provide artificial ventilation.

A thermometer will often have very good thermal contact with its supporting mount, which is a good conductor of heat. A good design provides sensor supports with low thermal conductivity and uses small connecting wires.

The temperature sensor is apt to be a good absorber of solar radiation and will almost certainly be a good emitter and absorber of terrestrial radiation. The ideal radiation shield would block the sensor view of the sun, sky, ground, and all surrounding objects. It would reflect all incident radiation and not reradiate to the sensor. The shield must also protect the sensor from precipitation. It should permit free natural ventilation and, if it cannot because of the above requirements, aspiration should be provided.

There are cases where no external radiation shield is used. If very fast response is desired, a small diameter polished wire sensor, such as a thermocouple, may be directly exposed to the atmosphere. A polished wire is a good reflector of radiation. A small diameter wire is a relatively poor heat conductor and has low thermal mass and therefore doesn't need much ventilation.

The amount of ventilation required is a function of the thermal mass, surface area, surface heat transfer coefficient, and the extent of secondary heat transfer mechanisms due to conduction along the mount and due to radiative transfer. For an ordinary glass thermometer, 3 m/s is adequate.

4.1.5 QUESTIONS AND PROBLEMS

1. How do you determine the nonlinearity of a sensor? Give a detailed procedure.
2. If the reference junction of a type T thermocouple is held at 50°C and the observed emf is -1246 μV , what is the temperature of the

sensing junction? Which law of thermocouple behavior must you invoke to solve this problem?

3. Fit a straight line to a type T thermocouple. Describe the method you use and list the data. What is the resulting maximum error in the -30 to 50°C range due to residual nonlinearity?

4. What is the maximum nonlinearity of the PRT described in Eq. 2, Table 4-3, and Figure 4-3? At what temperature does this occur? The nonlinearity can be expressed in terms of the raw sensor output, resistance, or in the equivalent temperature error. Do it both ways.

5. A simple linear approximation to a PRT can be obtained by ignoring the b and c terms in Eq. 2. Extend Table 4-3 by inserting resistance values obtained by assuming simple linearity. What is the maximum resistance error? Plot resistance error vs temperature. Can you convert this resistance error to equivalent temperature error? Plot it.

6. Plot the nonlinearity of the PRT, bridge circuit combination shown in Figs. 4-4 and 4-5.

7. How much current flows through the PRT in Fig. 4-4, assuming that the input reference voltage is 1.0 V and that $R = R_2 = R_3 = R_4 = 100 \Omega$? Is this enough to cause significant self-heating in an air temperature sensor?

8. Assume that resistors R_2 and R_3 have been removed from Fig. 4-7. Recall that $R_1 = 18700 \Omega$ and R_4 is the #2 thermistor from Table 4-4. Then, if the input voltage $E_1 = 1.00$ V, the output E_2 will be given by $E_2 = R_4/(R_1 + R_4)$. Assume that E_2 will be read by a digital voltmeter in the 1.00 V range, and that it has 3-digit resolution and accuracy. That is the reading must be expressed as three digits in the range $.000$ to $.999$. Compute the 3-digit reading at -30 and -29 , 0 and 1 , 49 and 50°C . For each pair of temperatures, compute the system sensitivity, i.e., the change in temperature corresponding to a change of reading of one count (change of $.001$ in the reading).

9. Repeat the above with the original circuit in Fig. 4-7.

10. Make a table of all the temperature sensors you can think of and list the pros and cons for each. Remember, more points for more sensors.

4.2 HUMIDITY

4.2.1 BACKGROUND

Introductory Comments

Humidity is one of the most difficult basic meteorological variables to measure. The vast array of different devices available is testimony to the continued search for a decent piece of humidity data. The sensitivity of virtually all practical field instruments to environmental contamination makes the quality of humidity measurements extremely technique dependent. The lack of an inexpensive, accurate standard is another unfortunate situation. The laboratory calibration of humidity sensors against the best, readily available commercial device (the cooled-mirror dew-point sensor) requires fairly complex temperature and water-vapor control machinery. Wexler (1970) is recommended for a more than detailed discussion of humidity measurement principles and problems.

Recommended Background

A general familiarity in basic physics and meteorology is suggested for students who are to be subjected to this material. Working knowledge at the level of Fleagle and Businger (1980) or Wallace and Hobbs (1977) is appropriate. A brief introduction to the primary subjects of interest can be found in the early parts of Chapter 6 of Fritschen and Gay (1979) or the (British) Meteorological Office Handbook of Meteorological Instruments, Volume 3 (1981) which is hereinafter referred to as BMOH#3. Some specific areas required are listed below:

1. The ideal gas law, physical properties of water vapor, and definitions of various humidity variables (dew-point temperature, mixing ratio, relative humidity, specific humidity, water-vapor density, etc.).
2. Thermodynamic and equilibrium properties of the liquid/vapor system, including latent heat, saturation vapor pressure, Clausius-Clapeyron relation, the Kelvin effect, the Raoult effect, and Kohler curves. The best source for advanced material on these subjects is Pruppacher and Klett (1978).
3. The study of instruments with application primarily in the realm of micrometeorology will require more advanced expertise on the order of the senior/graduate student. A good foundation in surface-layer scaling theory and turbulence can be found in the early chapters of Haugen (1973). A development with emphasis on humidity is given by Wyngaard et al. (1978), and Wyngaard and LeMone (1980). Key variables of interest are humidity flux, variance profile, gradient, power spectrum, and structure function parameter.

4.2.2 SENSOR PHYSICAL PRINCIPLES

The physical principles of humidity sensors have been outlined and discussed with considerable thoroughness by Wexler (1970). The important processes are listed below with a minimum of discussion.

Condensation

These sensors are based on the formation of dew when the sensor is at the temperature where the saturation vapor pressure is equal to the ambient partial pressure. Another approach is based on the change of physical properties of salt solutions at the saturation point (in this case the saturation-point temperature is a function of dew-point temperature).

Equilibrium Sorption

These sensors utilize the sensitivity of various physical properties with absorption of water vapor as a function of relative humidity. Some sample physical properties, particularly sensitive to water content, are electrolytic resistance, surface resistivity, bulk resistance, and mechanical dimension.

Extinction of Light

The absorption and scattering of light by air can be a strong function of water-vapor density and wavelength (e.g., Fleagle and Businger, 1980; Liou, 1980). The reduction of initial light intensity, I_0 , to a value, I , by a path length, x , is most simply given by the Beer-Lambert Law

$$I = I_0 \exp(-\alpha_\lambda \rho_v x),$$

where ρ_v is the water-vapor density, λ the wavelength and α_λ the extinction coefficient. See Buck (1983) for a discussion of broadband absorption and absorption by other atmospheric constituents in the ultraviolet. The major broadband effect is that α is a function of total absorber amount, $\rho_v x$. In the infrared wavelength range multiple, fine line structure must be considered, see Liou (1980) for information on water-vapor absorption lines in the IR.

Evaporation

The evaporation process involves the transfer of latent and sensible heat and water vapor from a ventilated wet object. Hinze (1959) and Schlichting (1979) are recommended for discussions and transfer coefficients. The idea is that the rate of evaporation from a sensor of length, ℓ , is simply expressed as a transfer coefficient, N_e , and the water-vapor diffusivity, K_e , and the difference in the vapor pressure of the wet object and the partial pressure of the ambient air

$$F_e = \pi \ell K_e N_e (\rho_s T_{wet} - \rho_v),$$

where N_e is a function of ventilation speed, u . Similarly, the flux of sensible heat to the wet bulb, which is required to balance the latent heat absorbed by evaporation, can be expressed

$$F_s = \pi \ell K_a Nu (T_{dry} - T_{wet}),$$

where K_a is the heat diffusivity of air and Nu the Nusselt number. Wexler (1970) and BMOH#3 provide some background on this subject.

Refractive Index

In radio and microwave frequency ranges, the refractive index of electromagnetic waves in air, N , is a function of temperature, T , pressure, p (mb), and specific humidity, Q (g/kg). To a good approximation Friehe, (1977); Weseley, (1976); Dobson, Hasse and Davis, (1980) the dependence is given by

$$Q = \frac{AT^2}{p} N - BT,$$

where $A = 1.67 \times 10^{-3}$ and $B = 0.129$.

4.2.3 SENSOR DESCRIPTIONS

Background

There are many different humidity sensors in use today. Since most of them are adequately described elsewhere, there is little to be gained by writing a lengthy treatise here. For the purpose of general instruction on these devices, the existing literature will simply be cited, and it is left to the instructor to get up to speed. Fritschen and Gay (1979) provide reasonable material on many equilibrium sorption sensors. While BMOH#3 is excellent for psychrometers and the hair hygrometer, Dobson, Hasse, and Davis (1980) in Chapters 21 and 22 provide good material on the remaining devices. For an examination of the accuracy of many commercial devices, see McKay (1978).

Condensation

Cooled-mirror dew-point heated LiCl solution (dewcell).

Equilibrium Sorption

Hair, LiCl (Dunmore), Brady array, piezoelectric, thin film, carbon hygistor, aluminum oxide, Humicap.

Absorption of Light

Lyman-alpha (uv) and various IR wavelengths.

PsychrometersMicrowave refractometer

4.2.4 APPLICATIONS

Examples

A few examples of humidity sensor utilization in meteorology are given below. Those measurements requiring fast response sensors (absorption of light, psychrometers, and microwave refractometer in Section 4.2.3) are indicated with *.

1. Fluxes*, turbulence variables* (Lenschow, Wyngaard, and Pennel, 1980).
2. Atmospheric radiation (Fleagle and Businger, 1980).
3. Aerosol properties (Fitzgerald, 1975).
4. Clouds, fog (Gerber, 1980).
5. EM propagation* (Fairall, Davidson, and Schacher, 1982).
6. Boundary-layer dynamics (Davidson et al., 1983).

Sensor Performance

The selection of a particular sensor involves consideration of the application and a variety of sensor performance factors. Wexler (1970), McKay (1978), and Dobson, Hasse, and Davis (1980) provide much information on sensor performance. One of the major problems in evaluating humidity sensors is the lack of a truly suitable secondary standard for field work (see Section 3.4.4). This role has fallen to the cooled-mirror dew-point sensor, which under laboratory conditions is good to a few tenths of a degree centigrade. Table 4-5 provides examples of some relevant performance criteria and some crude estimates of typical values for each sensor class. The accuracy of humidity sensors, being fairly dependent upon the operator's ability to prevent or repair contamination, is a somewhat nebulous concept.

Table 4-5. Relevant performance criteria of sensors.

Criterion	Device Class				
	Condensation (Td)	Sorption (RH)	Extinction (ρ_v)	Psychrometer (RH)	Refractometer (Q)
Cost* (\$)	5000	100-1000	2000-10,000	150-1000	10,000
Accuracy	0.5°C	5-15%	5%	3%	0.5 g/m ³
Time Const. (S)	10	10-500	0.01	0.1-30	0.01
Useful Range (RH)	5-100	20-95	0-100	20-100	20-100
Liq. Water Tolerance	excellent	poor	fair/good	good	good
Field Maintenance	low	low	high	moderate	low
Typical Application	cal. std.	radiosonde	flux	profile	structure function

*Approximate for full system.

+Assuming water is removed from the sensor surface.

4.2.5 QUESTIONS, PROBLEMS, AND LABORATORY EXERCISES

Questions

1. Is moist air "heavier" than dry air?
2. The addition of salt lowers the vapor pressure of water. What type of chemical could increase the vapor pressure?
3. The transition from vapor to liquid (and vice versa) is a first-order thermodynamic process. What does this imply about devices that use condensation, evaporation, and hygroscopic principles?
4. Suppose a humidity sensor is placed at the exhaust end of a long tube. What water-vapor properties can we deduce about the air when it first entered the intake of the tube? What properties cannot be deduced without additional information? Elaborate on possible errors introduced by this method.
5. A cooled-mirror dew-point device has a dew "thickness" adjustment. Should we increase or decrease the thickness to reduce salt contamination errors?
6. Hygroscopic instruments must be protected from liquid water. Discuss the ramifications of enclosing the sensor in a membrane that is permeable to water vapor but impermeable to liquid water.
7. The psychrometer uses the latent heat of vaporization of water to cool a wet-bulb thermometer. In this case, water molecules go from the wet bulb to the air. Suppose one wets the bulb with a liquid that absorbs water vapor. Would the "wet" bulb warm or cool? Discuss the advantages, disadvantages of such a system.
8. Why does a psychrometer become insensitive to ventilation at higher speeds? Discuss the intelligence of sticking a sling psychrometer out the window of an airplane to measure relative humidity?
9. Why is it a poor idea to keep a cooled-mirror dew-point sensor too clean?
10. Why do hair hygrometers use human hair?
11. Why would the use of EM-absorption type sensors on an aircraft require more thorough calibration than ground based applications?
12. Discuss the relative accuracy of a microwave refractometer in the tropics vs the arctic.
13. What sensor characteristics would you emphasize for profile measurements? Turbulence? Unattended buoys?

14. What class of sensor would be more appropriate to study meteorological factors affecting a) visibility, b) crop wilting, c) lifting condensation level, d) photochemical smog, e) thermal plumes, and f) power plant cooling-tower efficiency.

Problems

1. Calculate the equivalent uncertainty in relative humidity as a function of air temperature (0°C to 30°C) for a dew-point temperature error of 0.5°C.
2. Joe Blow is insufficiently erudite to realize that the $T_d = -10^\circ\text{C}$ reading from his cooled mirror dew-point sensor is not the dew point but actually the ice point. How great an error does he make in calculation of vapor density?
3. The calibrated output of a Lyman-alpha system is high-pass filtered at a frequency, f_l , and low-pass filtered at a frequency, f_u . Assuming the inertial subrange power spectrum form

$$\phi_q(k) = 0.25 C_Q^2 k^{-5/3},$$

where $k = 2\pi f/\bar{u}$ for frequency, f , and mean relative flow speed, \bar{u} , derive an equation that relates C_Q^2 to the measured water-vapor variance in the bandpass region.

4. A psychrometer is ventilated at 5 m/s. The albedo of the dry bulb is 0.9, while the albedo of the wet bulb is 0.5. Calculate the error in wet-bulb depression and relative humidity if the unsheltered sensors are exposed to an insolation of 1000 W/m²? Assume the sensors are cylinders of 0.5-cm diameter and 3-cm length.
5. Assuming a well-mixed (no vertical gradient) water vapor mixing ratio and potential temperature, use the Clausius-Clapeyron equation to calculate the surface lapse rate of dew-point temperature.
6. To what accuracy can one measure the water-vapor surface-layer scaling parameter

$$q_* = kz \left(\frac{\partial q}{\partial z} \right)$$

under neutral conditions with two cooled-mirror dew-point devices at heights z_1 and z_2 ($k = 0.4$). Assuming $z_1 = 5$ m, $z_2 = 15$ m and $u_* = 30$ cm/s, express this uncertainty as a rain rate equivalent in mm/hr.

7. A cooled-mirror (mass, M) dew-point system uses a thermoelectric cooler with maximum cooling rate, W . The cooler output is controlled by a circuit that uses the integral of the sensor temperature, T_s , dew-point temperature, T_d , error signal (negative means cooling)

$$H_c = -GW$$

where

$$G = \frac{1}{T_0} \int (T_s - T_d) dt$$

for some scale factor T_0 and $0 < G < 1$. The ventilation of the mirror causes it to be heated according to

$$H_v = \frac{\alpha W}{T_0} (T - T_s)$$

where α is a constant and T the ambient air temperature. Derive the differential equation for the system dynamic response (hint: take the derivative of the heat balance to eliminate the integral form of H_c). What is the resonant frequency, damping factor, and maximum measurable dew-point depression?

Laboratory Exercises

Laboratory Exercise #1: Humidity--Sensitivity of Psychrometry

Find the errors in dew point, relative humidity, and mixing ratio associated with making a 0.5°C error in dry-bulb temperature, a 0.5°C error in wet-bulb temperature, and a 0.5°C error in wet-bulb depression when:

1. The temperature is 20°C and there is 95% relative humidity.
2. The temperature is -20°C and there is 95% relative humidity.
3. The temperature is 20°C and the relative humidity is 20%.

Laboratory Exercise #2: Dewpointer Calibration

Purpose:

To check the calibration of a cooled-mirror dewpointer and observe some operating characteristics which cause difficulty. This experiment can be run simultaneously with the thermometer calibration experiment.

Equipment:

Cooled-mirror dewpointer, variable temperature bath, copper coil, and aspiration pump.

Procedure:

Air circulated through the coil in the temperature bath emerges with a dew point equal to the bath temperature, providing sufficient moisture is present in the closed system, and there are no moisture sinks. Compare the dewpointer readings with the bath temperature readings and observe the dewpointer behavior as temperatures enter the frost-point region.

Results:

Tabulate the differences between the two instruments and note the characteristics of the behavior at decreasing temperatures.

Question:

The cooled-mirror dewpointer is sometimes billed as a "primary standard." In what sense(s) is this true, and why might such a designation be disputed?

Laboratory Exercise #3: Humidity fixed points.**Purpose:**

To demonstrate the use of saturated salt solutions as humidity generators. To calibrate a carbon hygistor.

Equipment:

Prepared saturated salt solutions, carbon hygistor, and digital volt-ohmmeter.

Procedure:

The "lock-in" resistance of a carbon hygistor is its resistance at 33% RH, and humidity measurements are usually made by use of the ratio of hygistor resistance to the lock-in resistance. The lock-in resistance may drift grossly, while the use of resistance ratio still provides an acceptable measurement.

Determine the lock-in resistance, then the resistances at other humidities. Finally, recheck the lock-in resistance.

Results:

Display the calibration results as a table of resistance ratios vs relative humidity, or as a calibration curve.

Questions:

1. Suppose the second determination of lock-in resistance was significantly different from the first. What causes and remedies might you suggest?
2. The carbon hygistor has significant temperature effects, none of which are evident in this calibration at a single temperature. What temperature effects would affect the measurements in radiosonde use, and how might they be corrected?
3. List some cautions in the use of the saturated salt solutions.

4.3 WIND

4.3.1 REQUIRED BACKGROUND

Certain aspects of sensor characteristics can be understood with a minimum of mathematical and physical understanding. In general, however, a calculus-level course in university physics and mathematics is necessary. However a separate course in differential equations, for a full understanding of underlying principles, is also necessary. Although specific training in meteorology is not essential (a junior level physics or engineering student would not experience deficiencies), a course in meteorology at the level of Wallace and Hobbs (1977) will provide the level of sophistication in problem solving necessary to master the material included.

4.3.2 THEORETICAL BACKGROUND

Dynamic Equation for a Rotating Anemometer

To understand the dynamics of a rotation anemometer (cup or propellor), we must examine a simple mathematical model as described by Wyngaard (1981). Consider the two-cup configuration shown in Fig. 4-10, which has the essential features for developing such a simple model for both general multiple-cup and propellor anemometers. Let the ambient wind speed be U and the linear speed of the cups (center) be S . The angular speed is therefore $\Omega = S/r$, where r is the distance of the center of the cup from the axis of rotation. The wind has speed $U - S$ relative to the right cup for the angular position shown.

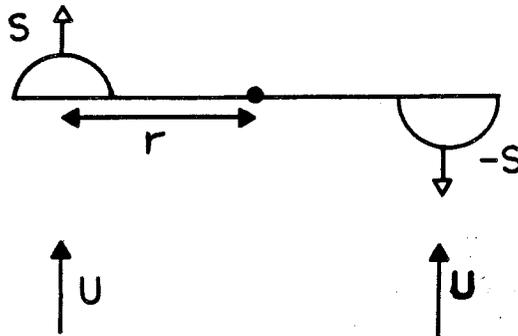


Fig. 4-10. Two-cup configuration.

The cup wheel moves in the clockwise direction because the cup geometry is such that the wind creates a greater force on the left cup than on the right. The force exerted by the wind on the left cup is given (Landau and Lifshitz, 1959) by

$$F_{\ell} = 1/2 \rho A c_{\ell} u_{\ell}^2, \quad (1)$$

where ρ is the air density, A is the cup cross-sectional area, c_{ℓ} is the drag coefficient for the left cup, and u_{ℓ} is the wind speed relative to the left cup. The torque about the rotation axis created by the left cup is then $\vec{T} = \vec{r} \times \vec{F} = 1/2 \rho A c_{\ell} u_{\ell}^2 \vec{t}$ for the position shown, where \vec{t} is the unit vector into the page. Similarly, for the right cup, $\vec{T} = -1/2 \rho A c_r u_r^2 \vec{t}$, thereby giving a net torque of magnitude

$$T = 1/2 \rho A r [c_{\ell} u_{\ell}^2 - c_r u_r^2]. \quad (2)$$

For the simple configuration of Fig. 4-10, the torque will be a function of rotation angle. Brevoort and Joyner (1935) (Slade, 1968) showed that the maximum torque produced by a single cup occurs when the concave side of the cup makes an angle of about 45° with respect to the wind direction and minimum torque at about 100° . It is observed that three cups give a more uniform torque delivery than either two or four cups (Patterson, 1926). Lockhart (1978) has shown that for a three-cup configuration, the torque extrema occur when one cup arm is orthogonal to the wind: the maximum torque occurs when a single cup rotates into the wind and minimum when a single cup rotates with the wind. We will ignore the angular dependence of the torque and consider rather the fluctuations in torque arising from wind speed and cup speed fluctuations.

Let $U = U_0 + u$ and $S = S_0 + s$, where U_0 and S_0 are mean values, and u and s are fluctuations. In the absence of fluctuations, the torque is zero and, hence,

$$T = 1/2 \rho A r [c_{\ell} (U_0 - S_0)^2 - c_r (U_0 + S_0)^2] = 0, \quad (3)$$

which gives

$$c_{\ell} (U_0 - S_0)^2 = c_r (U_0 + S_0)^2. \quad (4)$$

Under these conditions, the cup speed is observed to be a fraction, K (typically 0.3), of the wind speed, so we set $S_0 = KU_0$. For fluctuating wind conditions we then have

$$C_{\lambda} U_0^2 (1 - K)^2 = C_r U_0^2 (1 + K)^2, \quad (5)$$

and a torque

$$\begin{aligned} T &= 1/2 \rho A r [C_{\lambda} (U_0 + u - S_0 - s)^2 - C_r (U_0 + u + S_0 + s)^2] \\ &= \frac{2\rho r C_{\lambda} A K U_0^2}{(1 + K)^2} \left[(1 - K^2) \frac{u}{U_0} - (1 - K^2) \frac{s}{S_0} + \left(\frac{u}{U_0}\right)^2 \right. \\ &\quad \left. + K \left(\frac{s}{S_0}\right)^2 - (1 + K^2) \frac{s}{S_0} \frac{u}{U_0} \right]. \end{aligned} \quad (6)$$

The torque is related to the change in angular velocity by

$$T = I \frac{d\Omega}{dt} = \frac{I}{r} \frac{ds}{dt}, \quad (7)$$

which, together with Eq. 6, gives the dynamic equation for the cups. This equation now is of the form

$$A \frac{d\vec{s}}{dt} + B\vec{s} + C\vec{u} + D\vec{u}\vec{s} + E\vec{u}^2 = 0, \quad (8)$$

where \vec{u} is the dimensionless applied signal, u/U_0 , and \vec{s} is the dimensionless system response, s/U_0 . The last two terms on the LHS make this a second-order equation, creating a significant complication to analysis of the general system.

We can, however, gain insight to anemometer response by making the simplifying assumption that $u \ll U_0$ and $s \ll U_0$, thereby making the nonlinear terms very small compared to other terms of the equation. The anemometer equation is then

$$A \frac{d\vec{s}}{dt} + B\vec{s} = c' \vec{u}, \quad (9)$$

or

$$\frac{L}{U_0} \frac{ds}{dt} + S = Ku, \quad (10)$$

where L is the anemometer distance constant given by

$$L = \frac{I}{\rho r^2 CA}, \quad (11)$$

where C is an effective drag coefficient. A time constant, τ , can then be defined by

$$\tau = \frac{U_0}{L}. \quad (12)$$

For a step change of input speed from $u = 0$ to $u = u_a$, the solution to Eq. (10) then is

$$s = u_a (1 - e^{-t/\tau}), \quad (13)$$

which shows that, as with a temperature sensor, the anemometer reaches 63.2% of the new equilibrium value in time, τ . It should be noted that τ is a function of mean wind speed, U_0 , whereas L depends only on anemometer geometry and air density.

If the wind speed fluctuations are time dependent, the solution of Eq. (10) is more complex. For a sinusoidal fluctuation $u_a(t) = u_0 \sin \omega t$, the solution is (Fritschen and Gay, 1979):

$$s = \frac{u_0 \omega \tau e^{-t/\tau}}{(1 + \omega \tau)^{1/2}} + \frac{u_0}{[1 + (\omega \tau)^2]^{1/2}} \sin(\omega t - \phi), \quad (14)$$

where ω is the angular frequency of oscillation of the cup speed and $\phi = \tan^{-1} \omega \tau$. At a sufficient time ($t \gg \tau$), the amplitude of the oscillations of the linear cup speed is

$$s_0 = \frac{u_0}{[1 + (\omega \tau)^2]^{1/2}}. \quad (15)$$

Consider now the response to a sudden change, u , in the wind speed from an initially equilibrium state ($s = 0$). The torque is given (from Eq. 6) by

$$\frac{I}{r} \frac{ds}{dt} = T = \frac{2\rho r c_{\lambda} A K U_0^2}{(1 + K)^2} [(1 - K^2) \frac{u}{U_0} + (\frac{u}{U_0})^2]. \quad (16)$$

Note that a gust ($u > 0$) gives a positive torque of greater magnitude than the negative torque given by a lull ($u < 0$) of comparable size, because the term $(u/U_0)^2$ gives a positive contribution to the torque

in both cases. This nonlinear term in the torque equation leads to an inflated mean torque and hence, an overestimate of the wind speed. Small fluctuations ($u \ll U_0$) give $(u/U_0)^2 \ll u/U_0$, so overspeeding by this mechanism is minimized in steady winds but may be large in gusty winds. Filtering the output signal will not eliminate this effect.

Kaganov and Yaglom (1976) studied the mean overspeeding of a cup anemometer due to fluctuations in both horizontal and vertical wind components in the atmospheric surface layer. From their analysis, they conclude that overspeeding errors caused by fluctuations in the vertical velocity do not exceed 1% for stable stratification. They estimate the error to be 1% to 3% near neutral and 1% to 6% for unstable conditions. Errors introduced by horizontal fluctuations are of the same sign as those arising from vertical fluctuations, so it is plausible that total errors from turbulent fluctuations could be of the order of 8% to 10%. They conclude that the overspeeding observations of Izumi and Barad (1970), shown in Fig. 4-11, could be explained by three-dimensional turbulent wind-speed fluctuations. Vertical wind fluctuations do not cause overspeeding in the horizontal axis propeller anemometers.

Busch and Kristensen (1976) report that overspeeding of a cup anemometer is dependent on the ratio of the distance constant of the anemometer to the roughness height and also on the ratio of the measurement height to the roughness height. Francey and Sahashi (1982) compared measurements of a staggered 6-cup anemometer with simultaneous observations with a sonic anemometer and found an overspeeding of $3.7 \pm 2.3\%$ for an anemometer height of 4 m over land.

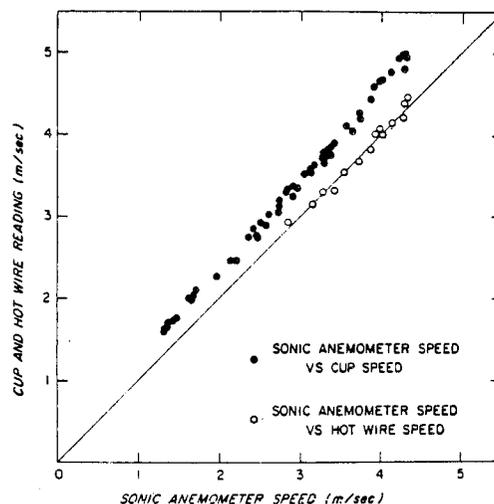


Fig. 4-11. Overspeeding observations of Izumi and Barad (1970).

Cup anemometers are also subject to overspeeding from exposure to winds having a constant nonzero component perpendicular to the plane of the cup wheel. MacCready (1966) shows that this overspeeding can be as much as 50% for certain anemometers and large angles with respect to the plane of the cups.

Dynamic Equation for a Wind Vane

The development from first principles of a torque equation for a wind vane is nontrivial. Several authors (e.g., Wieringa, 1967; Fritschen and Gay, 1979; Finkelstein, 1981) refer to the torque equation

$$N\beta = rF \quad (17)$$

where N is the torque per unit angle about the vane shaft, r is the distance from the rotation axis to the effective force point on the vane, and β is the angle between the wind and the vane. [Finkelstein (1981) erroneously indicates that the torque and force of Eq. 17 have the same direction]. Barthelt and Ruppertsberg (1957) present a derivation (in German) of this result generalized to a double fin configuration. A damping of the vane motion occurs due to air resistance and has direction opposite to that of the motion. If we assume small angular displacements, β , then the viscous damping torque is given by

$$\frac{Nr}{u} \frac{d\beta}{dt}; \quad (18)$$

where u is the wind speed. The equation of motion for the vane is then

$$I \frac{d^2\beta}{dt^2} = -N\beta - \frac{Nr}{u} \frac{d\beta}{dt}, \quad (19)$$

where I is the moment of inertia and $d = Nr/u$ is the damping force acting on the vane. The solution (Wieringa, 1967) is

$$\beta = \beta_0 \exp \left[-\frac{d}{2I} t - 2\pi i \frac{t}{t_d} \right], \quad (20)$$

where

$$t_d = 2\pi \left[\frac{N}{I} - \left(\frac{d}{2I} \right)^2 \right]^{-1/2} \quad (21)$$

is the damping oscillation period. The natural period is

$$t_0 = 2\pi \left(\frac{N}{I} \right)^{-1/2}. \quad (22)$$

The vane is overdamped if $\left(\frac{d}{2I}\right)^2 > \frac{N}{I}$
 critically damped if $\left(\frac{d}{2I}\right)^2 = \frac{N}{I}$
 underdamped if $\left(\frac{d}{2I}\right)^2 < \frac{N}{I}$.

Fritschen and Gay (1979) plot the response of vanes of various damping characteristics.

Vanes having double fins are discussed by Wieringa (1967). Some such vanes have more than one equilibrium position and are therefore unreliable for measuring wind speed.

Sonic Anemometry Equation

Kaimal (1980) has given a survey of sonic anemometry. Wyngaard (1981) summarizes the salient features of the technique. Fig. 4-12 (from Kaimal, 1980) shows the acoustic paths taken by pulses traveling in opposite directions. The transit times are

$$t_1 = \frac{d}{c \cos \alpha + V_d}$$

$$t_2 = \frac{d}{c \cos \alpha - V_d}$$

(23)

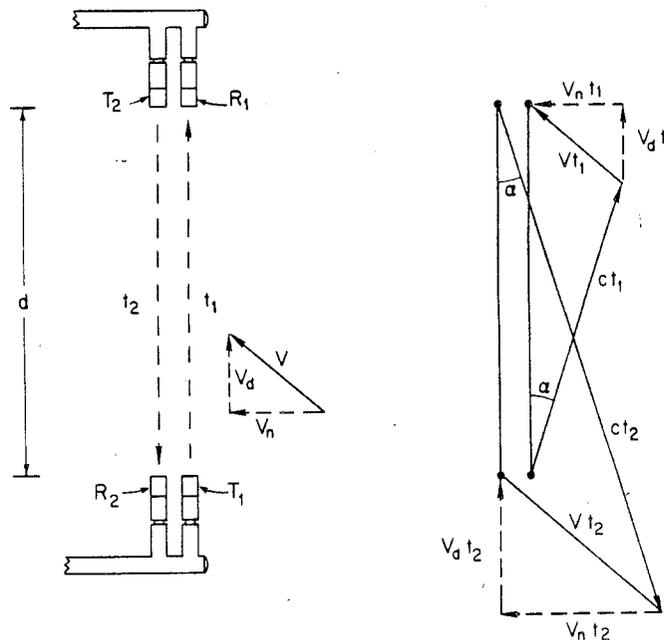


Fig. 4-12. Acoustic paths taken by pulses traveling in opposite directions.

where c is the velocity of sound and other symbols are defined in the figure. The time difference is then

$$\Delta t = \frac{2d}{c} V_d . \quad (24)$$

The wind speed component along the path, V_d , can be calculated from a measurement of Δt , and the mean temperature (necessary to calculate $c = \sqrt{\gamma RT}$, where γ is the ratio of specific heats and R is the gas constant). A more recent configuration of the three orthogonal transmitter/receiver pairs is shown in Kaimal and Gaynor (1983).

Elementary Turbulence and Boundary-Layer Theory

An introduction to at least elementary concepts of turbulence theory is essential for an understanding of the wide ranging effects of wind speed fluctuations. Short-term deviations of total wind vector from its time-averaged mean create the turbulent fluxes of momentum, thermal energy, water vapor, and air pollution that give the atmospheric boundary layer its unique characteristics. The standard meteorological literature provides descriptions of turbulence at a variety of levels. Sutton's text (1953) historically has been a standard reference, which, although it does not contain any of the myriad of advances of the last 30 years, contains the fundamentals that would provide the framework for an undergraduate level course. Tennekes and Lumley (1972) provides a good discussion for boundary-layer turbulence. Other possible texts include Holton (1979), Reynolds (1974), Hinze (1959), Haugen (1973), and Lumley and Panofsky (1964).

Several authors also have presented detailed descriptions of the motions and physical processes of the atmospheric planetary boundary layer (PBL) (Haugen, 1973; Holton, 1979; Fleagle and Businger, 1980; Wyngaard, 1984; to mention a few). I will present a very brief description of the qualitative features of the boundary layer as they pertain to wind measurements and refer the reader to the above listed sources for more detail.

The height of the PBL varies from a few hundred meters at night to 1 to 3 km by late afternoon as shown in Fig. 4-13. Maximum daytime height is much larger in summer than winter. The relationship of the wind speed and temperature profiles to the inversion height and diurnal pattern of surface heating also is shown in Fig. 4-13 for an idealized cloud-free sky and flat terrain.

At night, radiative cooling at the surface leads to laminar flow near the ground, which decouples surface flow from flow aloft. Nighttime surface flow is then frequently established by local-scale pressure gradients arising from low-level horizontal temperature gradients that, in turn, are created by orographic features. Daytime surface flow, by contrast, is likely to more closely resemble flow

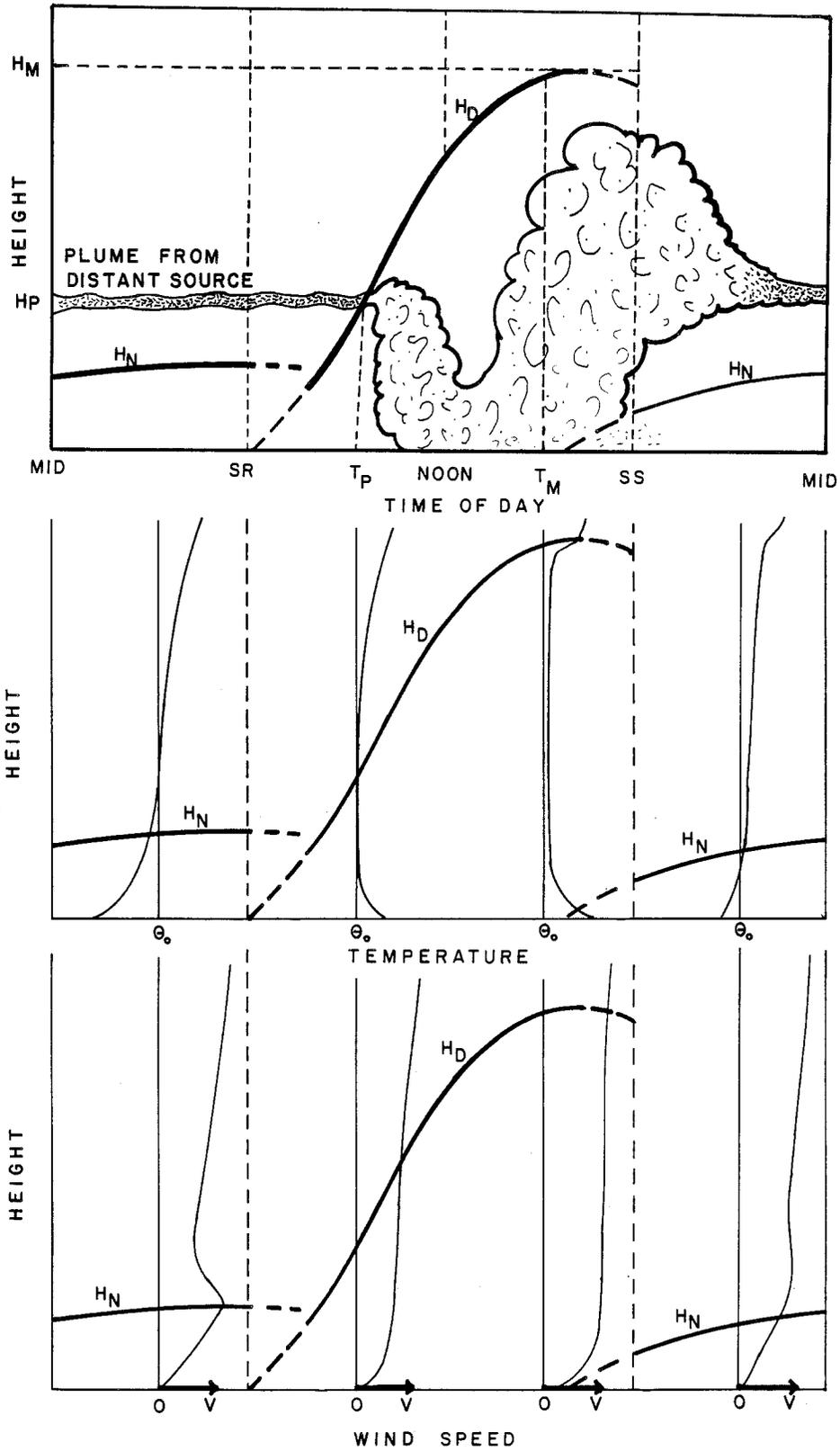


Fig. 4-13. Height variations of the PBL.

aloft because of the more direct linkage provided by convectively generated turbulence, which can carry information from 2 km to the surface with much greater speed and fidelity than is possible under the nocturnal regime. Large synoptic-scale pressure gradients will create winds that enhance this linkage, particularly at night. These factors, together with direct orographic influences, should be considered in evaluating the representativeness of a surface layer wind measurement.

The atmospheric surface layer, the lowest few tens of meters of the boundary layer, has some special properties. In this layer, the wind speed is observed to have a nearly logarithmic dependence on height. Deviation from purely logarithmic structure occurs for stable and unstable flow, as described by Haugen (1973). In the neutral case, the mean wind speed, u , is given by

$$u = \frac{u_*}{k} \ln \left(\frac{z}{z_0} \right), \quad (25)$$

where u_* = friction speed
 k = von Karman's constant, 0.4
 z_0 = roughness length .

A displacement height, d , is used if wind speeds over a forest, agricultural crop, or the like are being described (Molion and Moore, 1983), see Fig. 4-14. The velocity profile within the canopy is empirically described by an exponential function (Cionco, 1971; Cionco, 1972; Hayashi, 1983). If the flow field within the vegetation zone is not of interest, the mean wind profile may be described by

$$u = \frac{u_*}{k} \ln \left(\frac{z-d}{z_0} \right), \quad (26)$$

for $z > H$, as in Fig. 4-14.

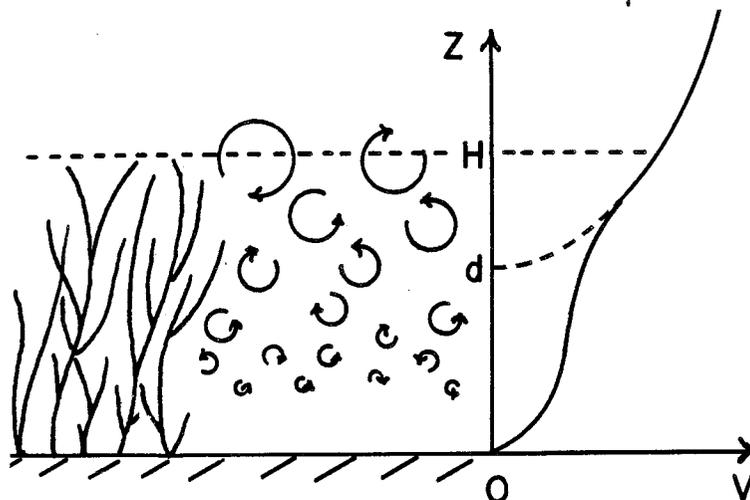


Fig. 4-14. Wind speeds over a forest.

An empirical power law also is sometimes used for estimating wind speeds at various levels:

$$\frac{u_2}{u_1} = \left(\frac{z_2}{z_1}\right)^n, \quad (27)$$

where u_2 is the wind speed at height z_2 , u_1 is the wind speed at height z_1 , and n is a number that depends on atmospheric stability. For neutral conditions, $n = 1/7$ (hence the common name "1/7 power law") has been observed to work quite well. For unstable, daytime conditions, $n = 0.10$ is more representative, whereas for stable flow, $n > 1/7$ and may be large and unpredictable because of the previously mentioned decoupling of the surface layer flow and the importance of local orography. Fig. 4-15 shows the diurnal and seasonal variations of the power law for a 32-m tower at a mid-continent location (Takle, Brown, and Davis, 1978). Although frequently used to estimate wind speeds above 100 m, the power law should be used with caution, particularly under stable conditions.

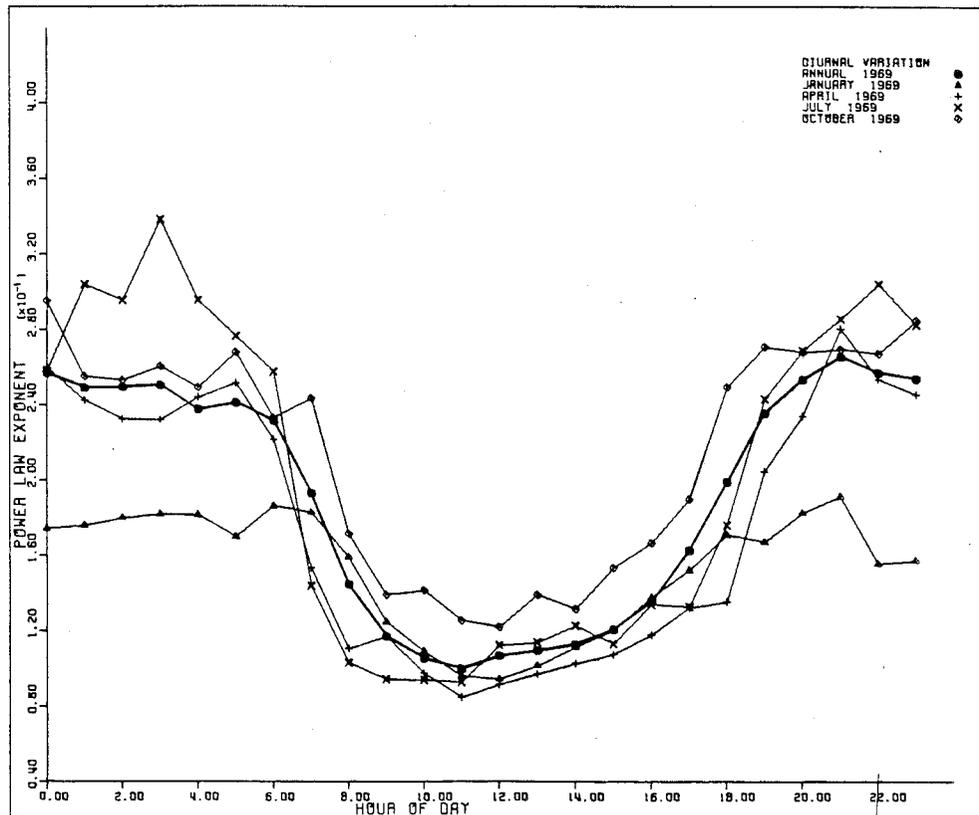


Fig. 4-15. Diurnal behavior of the power-law exponent, β .

4.3.3 DESCRIPTION AND CALIBRATION OF SENSORS

Cup Anemometers

The cup anemometer is the standard instrument for measuring wind speed in almost all operational surface meteorology and environmental measurement programs, because, compared to alternative sensors, it represents a balance of simplicity, sensitivity, durability, accuracy, and economy. If an anemometer is placed in a wind tunnel and its rotation speed is measured at several fixed values of tunnel wind speed, a linear relationship usually is observed over a range of approximately 1 - 100 m/s. This is a very desirable characteristic, because conversion of sensor output to a signal suitable for observation, recording, or processing is greatly simplified.

Cup anemometers are manufactured in a variety of geometries. The most commonly used configuration is a three-cup assembly with conical cups located on arms with centers at radius about 1.25 times the cup diameter. Patterson (1926) reported that the three-cup assembly delivered a more uniform torque than two or four cups. Two three-cup wheels mounted on the same shaft but with the two wheels 6° out of phase have been used in research studies by Franzen (1967) (Wyngaard, 1981) for more uniformity in torque. The maximum torque produced by a single cup occurs when the concave side of the cup makes an angle of about 45° with respect to the wind direction. Minimum torque occurs at about 100° . The variation in normal force coefficient with angle of attack is shown in Fig. 4-16 (Brevoort and Joyner, 1935; Slade, 1968).

Jones (1965) describes an anemometer as having 12 shallow conical cups on a single wheel. The large number of cups is needed to smooth the torque because of the fast response (0.1 s for a step change from 4 to 6 m/s). Scrase and Sheppard (1944) presented evidence that relatively lower levels of overspeeding in turbulent wind occurred for anemometers having conical cups as opposed to hemispherical cups. Deacon (1951) repeated the experiments and reported that very little difference was observed between the two-cup geometries and claimed the earlier results included instrument errors.

Angular speed of the anemometer shaft is determined by means of an on-off signal created by a switch or chopper or the voltage produced by an electrical generator. The most sensitive anemometers use a slotted disc or cylinder to interrupt a light beam to a photocell. This method extracts no energy from the rotating shaft.

A miniature dc generator connected to the shaft indicates wind speed directly by the voltage produced and requires no auxiliary counting or signal processing equipment to produce a reading of wind speed. Use of ac rather than dc generators avoids the need for brushes and a commutator, thereby reducing both friction and maintenance requirements. Permanent-magnet rotors with 2, 4, 6, or 8 poles are employed, the larger the number of poles, the more ac pulses per shaft

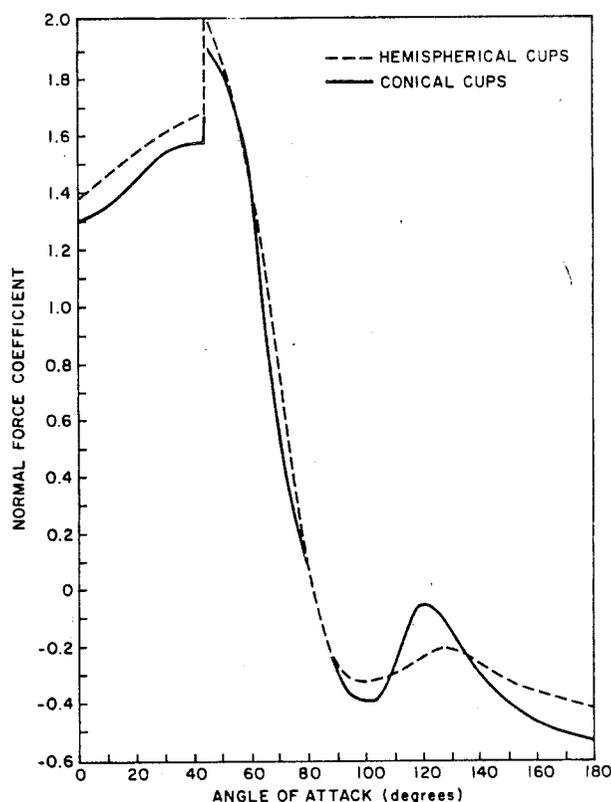


Fig. 4-16. Variation in normal force coefficients with angle of attack.

rotation. A frequency meter or rectifier is required to translate the signal. Wind speeds below 2 m/s require special correction if the sensor supplies a rectified ac signal.

Cup anemometers are calibrated by means of a variable-speed wind tunnel wherein the indicated speed can be compared with an independent sensor, usually a pitot tube. ASTM standards for calibrating cup and propeller anemometers are under development (ASTM, 1980a). However, they define the starting threshold as "the lowest speed at which a rotating anemometer continues to turn and produce a measurable signal when mounted in its normal position." This is not consistent with the conventional concept of starting threshold as the minimum speed required to initiate continuous motion for an anemometer at rest. The static friction of a cup wheel at rest must be overcome before the wheel will turn. Because the dynamic friction is less than the static friction, a moving cup wheel will continue to rotate at wind speeds less than those required to initiate cup-wheel movement, as shown in Fig. 4-17. In view of this ambiguity, it is important to clearly state how a measurement is made in reporting a threshold speed.

ASTM also has a procedure for measurement of an anemometer distance constant. A measurement is made of the time required for the anemometer rotor to accelerate $(1 - 1/e)$ or 63% of a step change in

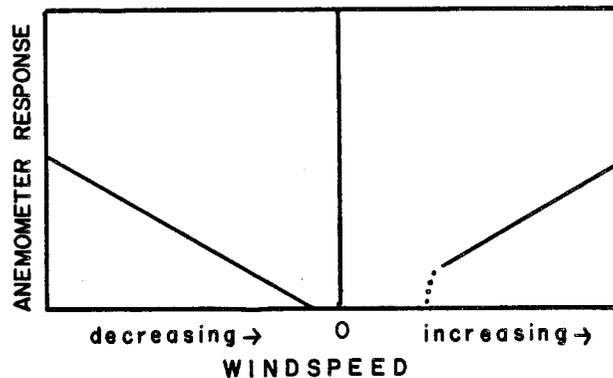


Fig. 4-17. Wind speed.

wind speed after release from a stalled, or nonrotating, condition. This measurement is made at between 30% and 74% of the wind-tunnel equilibrium speed in order to avoid the unrealistic effects of the stalled condition. This time in seconds is converted to the distance constant by multiplying by the tunnel-wind speed in m/s.

Propellor Anemometers

Propellor anemometers measure wind-speed component parallel to the shaft on which the blades are mounted. Two orthogonally mounted propellers (or one oriented by a vane as discussed later) are therefore required for a measurement of horizontal wind speed. A third propellor can be mounted on a vertical shaft to give the three-dimensional wind vector.

The blade tip speed of a propellor anemometer is typically four times as large as the cup speed of a cup anemometer and, therefore, can be made more sensitive to very light winds. The helicoidal design causes the propellor to give a response that varies approximately as the cosine of the angle from the propellor axis. Pond et al. (1979) reported observations of ratios of measured speed to speed along the propellor axis as a function of angle of the wind from the propellor axis. Their results, shown in Fig. 4-18, reveal that for angles within about 30° of the propellor axis, the response follows the cosine law reasonably well but underestimates the speed by 10 - 15% at 45° . In practice, this effect and a modest speed reduction, due to wind shadow from the mounting for winds striking the propellor "from behind," are typically accounted for by data processing in the micro-electronics in amounts determined by wind-tunnel calibration.

Sonic Anemometers

Sonic anemometry is becoming more widely used for turbulence research because it offers high frequency response characteristics, while at the same time it has a sufficiently rugged construction for

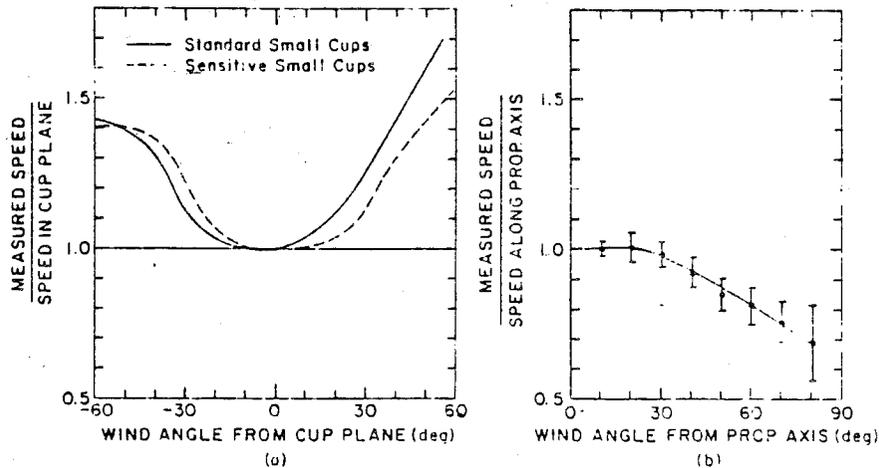


Fig. 4-18. Directional-response characteristics of typical cup and propeller anemometers: (a) cup response, from MacCready (1966), (b) Gill propeller response, from Pond et al. (1979).

continuous operation. Sonic anemometers typically are an order of magnitude more expensive than cup or propeller anemometers and require more sophisticated data handling capabilities. The data collection rate for a sonic anemometer typically is 200 measurements per second, thereby creating a large volume of data for a continuous operation mode. At the Boulder Atmospheric Observatory, (Kaimal and Gaynor, 1983), these data are used to form spectral and cospectral estimates averaged over 2-min intervals for the vertical velocity, whereas horizontal velocities are sampled in a slow response (1/s) mode and are retained as a time series.

Thermal Anemometers

Background. Thermal anemometers operate on the principle that the cooling rate of a warm object is a function of the relative wind speed. Such devices are useful in the laboratory where their small size (≈ 1 mm) provides good spatial resolution of mean flow patterns. They are not an "absolute" standard, so each sensor must be individually calibrated in a known flow. Since both static and dynamic calibration characteristics are unstable in contaminating environments, their use in field experiments is almost exclusively as high frequency velocity fluctuation sensors (where they excel). See Sandborn (1972) or Hasse and Dunckel (1980) for complete reviews.

Theory. In principle, the behavior of hot wires or hot films can be described by a simple heat budget that accounts for the various losses of heat from a resistive element (resistance, R) with a current, I ,

$$I^2 R = H_c + H_s + H_r + H_m, \quad (28)$$

where H_c is the forced convective loss, H_s is the wire support conductive loss, H_r is the radiative loss, and H_m is the rate of change of heat content. A good treatment of this subject can be found in Hinze (1959). The wind speed, u , enters through the forced convective term (assuming cylindrical geometry).

$$H_c = \pi \ell K N u \Delta T, \quad (29)$$

where ℓ is the sensor length, K is the thermal conductivity of air, ΔT is the sensor-air temperature difference (called the overheat), and Nu is the Nusselt number, which is given approximately by

$$Nu = C_1 + C_2 (ud)^{1/2}, \quad (30)$$

where C_1 and C_2 are properties of air, and d is the wire diameter.

Sensor description. The primary sensors of interest are hot wires and hot films. A typical wire (usually tungsten) is 5 μm in diameter and 1.5 mm in length. The wire is welded or soldered to two support posts that are connected to the power source. Hot film sensors consist of a thin layer of conductor (usually platinum) vacuum deposited on a quartz cylinder with diameter from 25 μm to 250 μm and length about 1.5 mm. The advantage of hot films is greater mechanical strength. A variety of more exotic physical configurations is available primarily for directional sensitivity (Hasse and Dunckel, 1980).

Operation. Hot-wire anemometers are operated in a constant temperature (or, more accurately, constant resistance) mode. A bridge circuit is used to pass sufficient current through the sensor to maintain its resistance at a preset value, R_s , which is larger than its resistance at ambient temperature, R_a . The overheat ratio is defined as R_s/R_a and is usually expressed as a percentage (50% is typical). Since the current required to maintain constant resistance is a function of wind speed, it is the signal used as the system output.

Characteristics. The static behavior of the hot wire is easily obtained (Let us ignore H_s and H_r) from the balance equation (note $H_m = 0$ for static equilibrium):

$$I^2 R_s = \pi \lambda \kappa K u [c_1 + c_2 (ud)^{1/2}] \frac{(R_s - R_a)}{\alpha R_a}, \quad (31)$$

where α is the temperature coefficient of the conductor, and

$$R_s = R_a (1 + \alpha \Delta T). \quad (32)$$

Thus, the relationship between sensor signal and wind speed (referred to as King's Law) is

$$I^2 = A + B \sqrt{u}, \quad (33)$$

where A and B can be obtained from the equations above. The dynamic characteristics require a consideration of all of the terms in the heat balance equation (see Larsen and Busch, 1974), but basically the conduction to the wire supports implies that a hot wire is a second-order system. Frequency response of hot wires is in the kHz range; whereas, for hot films, frequency is in the hundreds of hertz.

Calibration/Accuracy. Variations in calibration caused by chemical aging and environmental contamination affect both the static (Schacher and Fairall, 1976) and dynamic (Fairall and Schacher, 1977) accuracy of hot wires. The dynamic response often is determined in-situ by comparison at lower frequency with cup or propeller anemometers. Other factors that affect the accuracy are (Hasse and Dunckel, 1980):

- temperature fluctuations and mean variations
- orientation of the sensor
- lead resistance and capacitance
- rain
- radio interference.

Dynamic Pressure and Other Sensing Methods

Middleton and Spilhaus (1953) describe a variety of methods that historically have been used or proposed for measuring wind speed. The interested reader should check that reference for more details.

Pressure tube anemometers have long been used for wind speed measurements and are commonly used in wind tunnels. The pressure on a small opening either facing into or transverse to the wind is a

quadratic function of wind speed. A manometer reading of the pressure difference between two such openings can be calibrated to provide a measure of wind speed. A modern European version uses five holes in a sphere to create the pressure difference, thereby allowing observation of wind vector. Pressure plate anemometers also rely on this quadratic relationship of wind speed to force or pressure and have seen some limited use. Pressure plate anemometers suffer from difficulties of nonlinear drag coefficients and the necessity of being oriented into the wind.

The drag force, operating on a pedestal-mounted stationary sphere, has been sampled by use of strain gauge observations of pedestal motion. This configuration has the advantage of no moving parts but has had other problems that have prevented operational usage.

Dynamic pressure has been used to determine winds aloft by means of a kite connected by a nonstretching line to a load bridge/semiconductor strain gauge at the ground (Daniels and Oshiro, 1982a). Direction is sensed by two potentiometers fastened to the kite connecting line, so that both azimuth and elevation angles are determined. Other publications by the University of Hawaii Meteorology group (Daniels and Oshiro, 1980; Daniels and Oshiro, 1982b; Daniels and Oshiro, 1982c) describe data acquired by use of this system.

Bridled anemometers also sense wind stress by measuring the force required to prevent a multiple-cup wheel from turning. Bridled anemometers generally have a low sensitivity, are quite inaccurate at low speeds, and give a distorted response to wind gusts. This anemometer also is of purely historical interest.

Vortices produced by flow past a cylindrical obstruction have been used to determine wind speed by use of a vortex anemometer (Beadle, 1978). For a given obstruction, the wind speed is proportional to the frequency of vortex formation. An ultrasonic beam is used to detect vortex formation. These devices have been used in oceanographic application.

Wind Vanes

Most wind vanes in common use are single flat fins. The cross-wind shapes presently available seem to have arisen from aesthetic rather than aerodynamic consideration. Splayed (double fin) vanes are occasionally used but there seems to be no convincing rationale for the more complex geometry.

ASTM has established procedures for measuring the characteristics of a wind vane (ASTM, 1980b). Starting threshold is determined by measuring the lowest speed at which a vane released from a position 10° off the wind tunnel center line moves to within 5° of the center line. Delay distance is measured by observing the time required for the vane to reach 50% of the initial displacement of 10° off the wind tunnel

center line release. The delay distance, D , is calculated by multiplying this time by the wind-tunnel speed.

The overshoot is the maximum angular excursion of the opposite side of the initial 10° off-center line release point. The overshoot angle is divided by the initial (10°) displacement to give the overshoot ratio, Ω .

The damping ratio, η , is calculated from the overshoot ratio:

$$\eta = \frac{\ln(1/\Omega)}{[\pi^2 + (\ln(1/\Omega))^2]^{1/2}} \quad (34)$$

The damped natural wavelength, λ_d , is determined from

$$\lambda_d = \frac{D(6.0 - 2.4\eta)}{(1 - \eta^2)^{1/2}} \quad (35)$$

Bivanes

Three-dimensional wind direction can be assessed by use of a bivane, which consists of a bidirectional vane mounted on an arm that is balanced on a vertical shaft by a counterweight. Azimuthal direction information is transmitted by rotation of the vertical shaft to a potentiometer in the instrument housing. Elevation angular deflection is transmitted by a bead chain down the vertical shaft to a second potentiometer. A power supply and signal translator are required to convert the mechanical motions to analog signals that indicate azimuthal and elevation angles. Elevation range typically is $+50^\circ$ and the sensitivity approximately 0.1 - 0.2 m/s. Nominal values of delay distance, damped natural wavelength, and undamped natural wavelength are 1.0 m, 4.8 m, and 4.1 m, respectively. A damping ratio of 0.53 and maximum overshoot of 7% are typical.

Bivane use is typically restricted to short-term micrometeorological measurements. Vertical offsets due to precipitation, dew formation, dirt, birds, or insects limit their fidelity for continuous unattended operation. Nonzero mean vertical winds near the surface are not always spurious, however. Maitani (1983) observed that the mean vertical wind direction over a plant canopy was 3.2° rather than zero. Deardorff (1958) explains this phenomenon in terms of Reynolds stress and suggests the slope of the streamlines is related to the drag coefficient.

Vane Anemometers

Horizontal wind speed can be measured with a single propellor anemometer if it is mounted so as to be oriented into the wind by a vane. The propellor drives a miniature dc generator, which gives a

voltage output that is a linear function of wind speed. Shaft rotation gives wind direction through a potentiometer in the base. A power supply and signal translator are required to render sensor output signals suitable for display or recording.

A rugged version of the vane anemometer for use in winds to 90 m/s is the Aerovane. The heavier construction results in a higher starting speed (≈ 1.1 m/s) than for polystyrene blades (0.1-0.2 m/s) but triples the maximum wind that can be measured without sensor destruction.

Bivane Anemometers

Bivane anemometers permit measurements of three wind components with a single propellor mounted on a shaft, with a bivane to orient it into the wind. Very light materials, such as foam polystyrene, usually are used for propellor and vanes to provide the low inertia, fast-response characteristics necessary for this type of instrument. In the manufacturing of bivane anemometers, wind-measurement error is minimized by matching the phase lags of the vane and propellor. Analyses of the dynamics of a vane with a propellor are given by MacCready and Jex (1964) and Sanuki, Kimura, and Baba (1960).

Wind data collected by the bivane anemometer can be processed by a device called a sigma meter (Jones and Pasquill, 1959), which computes standard deviations of wind azimuth and elevation angles. These statistical data are used in turbulence, diffusion, and air pollution studies (Islitzer, 1961; Slade, 1968).

u, v, w Anemometers

Three propellor anemometers mounted on mutually orthogonal axes can be used to measure the three wind components separately. As opposed to a vaned anemometer whose propellor is always oriented into the wind and hence turns in only one direction, the u, v, w anemometer propellers are fixed in space and will rotate in the direction determined by the angle of attack of the wind. Each propellor drives a miniature dc motor whose voltage magnitude provides speed information, and voltage polarity is used to determine direction.

Sufficient separation of the individual sensors should be provided to minimize flow-field distortion. Wind-tunnel calibration should be used to correct for any response asymmetries introduced by reverse flow (air impinging on the propellor after having passed through the support assembly).

4.3.4 APPLICATION OF SENSORS

Examples of Use

A wind survey should always begin with a definition of the expectations and potential use of the collected data, with an emphasis

on the need for absolute and relative accuracy of the measurements. Only through such an analysis can we arrive at an optimal set of instruments and collect data of sufficient quality at minimum cost for procurement and maintenance. Some examples of wind surveys are listed below.

Boundary-layer studies. Anemometers with low starting thresholds are generally required. Several instruments on towers are often used, which necessitates frequent comparisons of calibration and starting thresholds. For studies that are of limited time duration, durability is less critical. If high frequency wind information is required, delicate instruments are needed, resulting in high maintenance costs. Wind-direction information usually is of less interest than speed data for boundary-layer studies in flat terrain. For studies in mountainous areas or when orographically-generated (e.g., drainage) flows are significant, sensitive wind-direction sensors are required.

Air quality investigations. Low thresholds and low-speed-calibration accuracy are extremely important, as estimated pollution concentrations are inversely proportional to the speed: a 1-mph error at a 2-mph speed results in a 100% error in concentration estimate. We often want to find the lowest 1-h mean speed during a measurement period, as this condition produces the highest pollution concentration. This, the highest concentration, is often the statistic used for air quality standards. Calibration errors at higher speeds are much less critical.

Wind-direction information is also very important for air pollution studies. Wind vane fluctuations sometimes are used as an indicator of turbulence characteristics (Islitzer, 1961; Slade, 1968; Sadharam and Murthy, 1983).

In air quality studies, it is frequently necessary to use wind data collected at a particular site and elevation above ground to make predictions at another location and/or elevation. The representativeness of a measurement is, in many studies, of more crucial concern than the accuracy.

Wind energy surveys. Wind speed below about 10-15 mph is totally unimportant for wind-power generation, so accuracy in this range is of little importance. At higher speeds, however, measurement accuracy is critical. Wind power is proportional to the cube of the wind speed, so a 22-mph wind can produce 33% more wind energy than a 20-mph wind. Economic projections of profitability require input of less than this level of uncertainty, so accurate wind-speed measurements are probably the most important factor in a wind-power project. Overspeeding can overestimate the wind power significantly. The instruments must be

carefully calibrated at higher speeds. Ability to withstand high and sustained winds is obviously required.

Long-term general purpose wind-speed monitoring. Ease of maintenance, ruggedness, and ability to maintain calibration are important factors for a long-term wind station. The limitations of the data must be kept in mind when the data are used for specific purposes.

Performance Criteria

After the intended use of the data is defined, we are ready to select the appropriate type of anemometer and recording system.

Types of anemometers.

Propellor anemometers. Fixed axis anemometers must be accurately oriented and their output compensated for noncosine responses. Propellor anemometers typically have more problems with corrosion than cup anemometers. Light, but fragile, propellers give lower starting threshold and more linear response at low speeds.

Vane anemometers. Problem with coupling of direction and speed fields.

Cup anemometers. Most common, no problem with orientation, and overspeeding can be significant.

Hot wires. Only for shorter, high-frequency measurement series, as they are very difficult to maintain.

Remote sensors. Laser anemometers, radar, pilot balloon, or radar generally used for specific research projects only.

Types of rotational motion transducers.

Generators. Ac (simpler) or dc (more common) calibration can change by unintentional blows which change the magnetic field strength of the permanent magnet. No external power source needed.

Light chopper. Keeps calibration better but needs external power source. More compatible with digital computer technology. Lower starting threshold as no magnetic field needs to be overcome. Sensors can be interchanged without recalibration.

Magnetic switches. Simple, but bouncing, switches can create double counting at higher speeds, rather uncommon.

Mechanical switches. Connected to either mechanical counters of electric pulse system. Mechanical wear and switch bouncing may create problems. Not commonly used today.

Strain gauges. Uncommon and difficult to use because of calibration drifts.

Types of recording.

Run of the wind. The total number of miles of wind that passes by the anemometer is recorded either by a counter read visually (e.g., daily) or on a strip chart read hourly. Strip chart records provide accurate hourly average speeds. Short period wind or gusts are not detectable.

Instantaneous recording on strip chart recorders. More common but less accurate average winds, as chart-reading bias can be serious. More time consuming to extrapolate but less electronics are required. Gust and short-period winds are detectable. Spring-wound recorders are available for sites without power. A simple electronic recording and rugged means of data recording.

Electronic recording. Probably the most common method today is digital conversion of sampled wind speed. The samples are either recorded directly on magnetic tape or are processed by a micro-processor, which can calculate summary statistics such as spectra or means and variances. These statistics are then recorded on tapes or in fixed memories, or are transmitted over telephone or satellite links. In general, as more electronics are required as compared with strip charts, it is advisable to use strip charts where electronic spare parts or trained personnel are hard to come by. It is the only practical way to collect data for turbulence analysis. Though practically feasible, analog recording on magnetic tape for subsequent digitizing is uncommon today.

Data Acquisition Strategies

Because of the relatively high cost of a wind survey, it is necessary to optimize the number of instruments and their locations and heights. As the wind field can vary considerably horizontally in complex terrain and generally increases significantly with height in the atmospheric boundary layer, more sensors are generally desirable than can be afforded. One must therefore make subjective judgment about the wind field and locate sites which are estimated to be representative of wind regimes in the area. Short-term mobile surveys can be useful for initial wind-field assessments. Erecting a tower taller than 30 ft is generally very expensive and requires more preparation and good accessibility. It might, however, still be advisable to go with a taller tower in order to monitor a more generalized wind flow, and be less influenced by vegetation, local surface conditions, and small hills. Kites (for high wind) or tethered balloons with anemometers suspended underneath (for low wind) provide a cheap and mobile method for shorter period measurements of upper-level winds.

Other important considerations in site selection are accessibility, power availability, security, protection against cattle, permits required, exposure, existing tower in generally less desirable locations nearby, existing quantitative or qualitative data from nearby sites, and physical or numerical model estimates for the area.

It is important to determine how the data are to be made available. Here it is advisable to consider the effects of data losses or inaccuracies. Can we live with data loss for a few weeks before a malfunctioning data logger is detected? Can we reprocess the data to correct instrument changes determined at a later calibration? The most common methods to store wind speeds are recording on magnetic tape or strip chart, calculating summary statistics on a microprocessor in the field and record in fixed memory or on magnetic tape, finding the total run of wind between site visits by merely subtracting a counter reading from the previous one, and telemetering the data on an hourly basis. Telemetry is probably the best way, as it provides almost instantaneous data which may be valuable during special weather conditions, such as storms where certain actions are taken as a result of the data. It also minimizes the time before malfunctions of the equipment are detected. Recording summary (e.g., hourly) data or statistics on tapes or fixed memories are generally cheaper and less complex, but it can result in long periods of data loss before malfunctions are found. Recording run of the wind between site visits is the cheapest and probably most reliable method of data collection, but the data have only limited usefulness as individual hourly readings are generally required for most purposes.

The frequency at which wind data are recorded depends on their intended uses as well as on instrument response and recording mode. Examples of wind speed data usage are as follows:

- Determining the long-term mean wind speeds. As there are generally only about two independent wind-speed readings per day twice daily, randomly sampled winds would be sufficient. It is, however, often difficult to assure random sampling which is necessary in this case.
- Short-term comparisons with other long-term sites. As hourly wind data generally are collected, hourly readings at the site are probably sufficient either taken as a sample on the hour or as a mean of several shorter term measurements.
- Special investigation of temporal cycles may require different recording frequency during the various times of the day.
- Turbulence studies requiring very frequent measurements. Wind speed should be sampled at least five times during the shortest time interval of interest. Instrument and recorder responses often limit the high-frequency end of the data.

If the data are to be archived for future nonproject specific use, hourly readings generally are sufficient. It is, though, important to document how and where the data were collected and what instrument and recording equipment were used. A description of calibration procedures and frequency and difficulty encountered should also be included.

4.3.5 QUESTIONS, PROBLEMS, AND LABORATORY EXERCISES

Questions

1. It was stated that one advantage of the cup anemometer is its linear response to wind speed; yet, it is a nonlinearity that can cause errors due to overspeeding. Explain.
2. Would you expect the relative error due to overspeeding to be larger for daytime or nighttime wind measurements? Explain.
3. Why do propellor anemometers rotate faster than cup anemometers of the same diameter (to effective force point) in the same wind? What does your answer suggest about a sailboat traveling directly downwind vs traveling at some acute angle with respect to the wind?
4. Why does a flag wave?

Problems

1. a) Show from Eq. 6 that if the mean wind speed, $U_0 = 0$, that

$$\frac{I}{r} \frac{ds}{dt} = \frac{\rho Ar}{2} [(s^2 + u^2)(c_\ell - c_r) - 2us(c_\ell - c_r)] .$$

- b) Show that if an anemometer at rest is given a fluctuation impulse of $u = At^2$, its initial response is $s = Bt^3$, where A and B are constants.
2. An anemometer with small sensitive cups has its axis of rotation mounted vertically (as it should be) on a ski slope of 30° inclination. Use the data of Fig. 4-18 to find
 - a) The true wind speed during the day when the anemometer reports an upslope wind speed of 10 m/s.
 - b) The true wind speed during the night when the anemometer reports a downslope wind speed of 10 m/s.

3. Estimates of wind speed at 100 m above ground (u_2) are needed for air pollution calculations for a power plant. Measurements of wind speed are made at 10 m (u_1) and the results extrapolated to 100 m by using the power law from Eq. 27. Compare the errors in u_2 due to calibration and sensor response (i.e., errors in u_1) with errors in u_2 attributable to representativeness (i.e., errors in the power law exponent, n) for daytime and nighttime, given the following information:

	u_1	δu_1	n	δn
Daytime	15.0 m/s	0.50 m/s	0.10	0.015
Nighttime	3.0 m/s	0.15 m/s	0.25	0.060

u_1 is the time averaged wind and δu_1 is the estimated error in u_1 ; n and δn , taken from Fig. 4-15, are the mean and estimated error in the power law exponent. Hint: consider $u_2 = u_2(u_1, n)$, and differentiate u_2 :

$$du_2 = \frac{\partial u_2}{\partial u_1} du_1 + \frac{\partial u_2}{\partial n} dn .$$

4. Wind power delivered from a wind-driven electrical generator is a cubic function of wind speed over the range between the threshold speed and 30 m/s. A generator and anemometer mounted at 50 m above the ground produce the following data:

V (m/s)	P (kW)
10	2.16
15	13.31
20	40.96
25	92.61

- Find the equation giving the generator output in kW as a function of wind speed in m/s.
- Your result can be written in a simple form, $P = A(V - V_0)^3$. Find A and the threshold velocity, V_0 .
- Now suppose the anemometer at generator level malfunctions, and, rather than replacing it at 50 m, the owner suggests that a second anemometer be mounted at 10 m and Eq. 27 and the equation for wind power be used to estimate power at 50 m from wind speeds measured at 10 m. Consider $P = P(u_1, n)$, and find the differential error in P resulting from errors in u_1 and n (see hint of previous problem).

- d) Use the wind speed and n values from the table in the previous problem to determine the error in wind speed at 10 m that will give errors in power of 25% if no error is made in the daytime value of n . Repeat for nighttime. Compare your results with δu_1 from the table of the previous problem.
- e) What error in daytime n gives power errors of 25% if no wind-speed errors are committed? Repeat for nighttime.
5. Derive the time difference equation for the sonic anemometer (Eq. 24) from Eq. 23 and Fig. 4-12 by assuming $V_d \ll c$.

Laboratory Exercises

Laboratory Exercise #1

Objective:

Determine distance constant of a cup anemometer.

Equipment:

Anemometer, fan with honeycomb or any other means to provide uniform flow across the anemometer cups at different speeds, high speed strip chart recorder.

Laboratory Exercise #2: Measurement of the Distance Constant of a Cup Anemometer

Objective:

The objective of this lab is to experimentally determine the distance constant of a cup anemometer.

Equipment:

R. M. Young Model 6101 cup anemometer, Sears two-speed fan, Varian potentiometric recorder.

Background:

In class we developed a linear model for the change of indicated wind speed with time (du/dt) as the anemometer responds to a sudden new wind speed U . As long as the difference ($U - u$) is small, the mathematical model is

$$\frac{du}{dt} = \frac{K}{T} (U - u) .$$

If an anemometer is initially in equilibrium with air speed U_1 and a step change in air speed occurs to U_2 , then, integrating the above equation between these limits results in

$$U_2 - u(t) = (U_2 - U_1) \exp \left(-\frac{K}{I} t \right) .$$

The term I/K is called the time constant τ but theory and experiment show that

$$\tau \propto \frac{1}{U_2} .$$

Further work has shown that the quantity

$$L = \tau U_2 ,$$

called the "distance constant" is essentially invariant for a given rotation anemometer. It has meaning only under the condition of a step change in wind speed. Also, there will surely be a problem if $U_2 = 0$.

In this lab we will simulate wind tunnel conditions that would ordinarily be used to determine the distance constant by using a fan to provide U_2 and holding the rotor of the cup wheel until release time so that $U_1 = 0$.

Procedure:

Before commencing the experiment select a partner. Each person will turn in their own lab report.

1. Turn on power to the recorder. Set the SPAN to X100 and SPAN mV/FS to 10. Set the CHART SPEED to 10 cm/min.
2. Turn on the fan to high speed and position the anemometer so that a comparatively steady output between 500 to 600 mV is observed.
3. Now you are ready to take a recording of data that will let you determine the distance constant for the R. M. Young Model 6101 cup anemometer.
4. One person hold the cup wheel rotor from below with two fingers so that there is no rotation. Make sure the output on the recorder is zero. After noting the position of the cups with respect to the air stream, release the rotor just when a time division line is being crossed. Obtain about 20 seconds of data.

You have just simulated the response of the cup anemometer to a step change in wind speed from zero to the equilibrium value on the chart. You will be able to determine the time constant and, knowing the wind speed, the distance constant.

5. With wind conditions being maintained, one person quickly and carefully place the cardboard box over the cup wheel. Do this when a time division line is being crossed, as observed by the other person (who then also turns off the fan). Record data sufficient to bring the output to zero (cup wheel is not turning).

You have just simulated the response of the cup anemometer to a step change in wind speed from the equilibrium value on the chart to zero wind speed. One can compute the time constant but not the distance constant (see Background).

6. Repeat steps 2 through 5 a sufficient number of times to get a total of 5 good measurements of the anemometer response to the upward step change in wind speed. A "good" measurement would be one in which the cup release was clean, the timing was correct, the wind speed was constant during the measurement, and the position of the cup wheel prior to release noted.

7. Repeat steps 2 through 6 except switch the fan to low speed and move the anemometer downstream from the fan so that a comparatively steady output of about 250 mV is observed.

8. When you are through, turn off the CHART, PEN LIFT, and POWER. Replace the pen cup. Tear off your portion of the chart paper.

Analysis:

1. Show on the chart paper the wind speed (in mV) and the time constant that will be used to calculate the distance constant for each test run. Of course, you will want to explain in your report how you selected the wind above and how you calculated the time constant.

2. Compare the average values of time constants for the four different cases (two for step change upward, two for step change downward). For example, for which case is the average time constant largest? smallest? Explain why. (Note: the required conversion factor for the anemometer is 2400 mV/50 mph).

For a given case of step change upward, what factors could account for the scatter among the time constants?

3. Compute the distance constant L (in feet) for each test run for the two cases of upward change. Plot the data on a graph of L vs U_2 (in ft/s and mph). Use, say, triangles and circles to distinguish the distance constants for each case.

Do the data show a dependence of L on wind speed? That is, is the difference between the mean values for the two cases large or small with respect to the scatter of data within each case?

What single value would you select as the distance constant for this anemometer? Explain. The manufacturer claims a distance constant of 12 feet. Do you believe it?

Laboratory Exercise #3: Calibration and Use of the Hot Wire Anemometer

Hot Wire Anemometer:

The hot wire anemometer is a device which may be described as a thermal anemometer. It measures wind velocity by: 1) measuring heat loss in a moving air stream by the body heated above the ambient air temperature, or 2) measuring the heat transfer from a heated body to a sensing element in close proximity to the heated body. Thermal anemometers range from the Kata Thermometer to highly complex hot wire and heated thermocouple anemometers. See Flow Corporation (1958), Hastings-Raydist, Inc. (1957), and Middleton and Spilhaus (1953).

Theoretically, any means of heating the anemometer above ambient air temperature might be used; however, electrical methods are almost always used for their convenience in application and measurement. The sensing element is nearly always electrical and may take the form of a thermocouple, thermistor, or metal wire.

One form of metal wire thermometer may be constructed by supplying the heating power directly to the sensing element.

Measurement of Parameters:

While it is obvious that there is a relationship between the heat lost from a body to a moving air stream and the velocity of the stream, the theoretical treatment of this relationship in the case of irregular and non-homogeneous bodies is rather complex and will not be discussed at this time. The case for a single heated wire has been approximated by King (1914) and will be used for this experiment.

By approximations and simplification, King concluded that the basic equation

$$W = B\sqrt{v} + C \quad (1)$$

applies for velocities normal to the wire, where W is the rate of heat loss per unit length of wire, v the air velocity, and B and C are constants dependent on the thermal characteristics of the air, temperature difference between air and wire, and the wire diameter. There is a limiting lower velocity for this expression which eliminates the simple case where C is the heat loss at v=0. This limit is

$$V \geq (0.0187) a \text{ cm/s}$$

where a is the wire diameter. However, if a is small enough, for wire smaller than 36 ga., the equation can be extrapolated to $v=0$, where $\Delta P'$ is the change in power (watts), E is EMF, I is the current, the subscript 0 indicates $v=0$, and the subscript v indicates $v>0$.

$$\Delta P' \cong W - C = B\sqrt{v} \quad (2)$$

$$\Delta P' = EI_v - EI_0 \quad (2a)$$

but

$$EI_v - EI_0 \cong B\sqrt{v} \quad (3)$$

Therefore,

$$\Delta P' \cong B\sqrt{v} \quad (4)$$

and

$$v \cong \left(\frac{\Delta P'}{B} \right)^2 \quad (4a)$$

EI_0 is the power required to maintain a constant temperature T in the heated wire when $v=0$. EI_v is the power required to maintain T when the wind speed is not zero.

The hot wire anemometer operates best when its temperature is such that the wire has a dull red glow. The element is heated to temperature T with $v=0$ to obtain the EI_0 values. More power is required to maintain the same intensity of glow of the hot wire as the wind speed increases, however

$$\frac{E_0}{I_0} = \frac{E_v}{I_v} \quad (5)$$

Thus, the ratio of EMF to current is the same regardless of wind speed providing the glow intensity is constant. A problem arises when adjusting the voltage and current at the higher wind speed and this

will create an error which must be corrected. In order to correct the error, the reference point error, ϵ , must be found by

$$\epsilon = \frac{\frac{E_0}{I_0} - \frac{E_V}{I_V}}{\frac{E_0}{I_0}} \quad (6)$$

Equation (6) may be solved for E_V and I_V

$$E_V = \frac{E_0}{I_0} (1 - \epsilon) I_V \quad (7)$$

and

$$I_V = \frac{E_V}{\frac{E_0}{I_0} (1 - \epsilon)} \quad (8)$$

A corrected expression for $\Delta P'$ can now be computed in three steps:

1) Assume that I_V is a correct value

$$E_V I_V = \frac{E_0}{I_0} (1 - \epsilon) I_V^2 \quad (9)$$

Then

$$\Delta P' = \frac{E_0}{I_0} (1 - \epsilon) I_V^2 - E_0 I_0 \quad \text{from (2a),} \quad (10)$$

$$\Delta P' = E_0 I_0 \left[(1 - \epsilon) \left(\frac{I_V}{I_0} \right)^2 - 1 \right] \quad (11)$$

- 2) Now assume that E_v is a correct value by the same process.
The final result is

$$\Delta P' = E_0 I_0 \left[\frac{1}{(1 - \epsilon)} \left(\frac{E_v}{E_0} \right)^2 - 1 \right]. \quad (12)$$

- 3) Since both 1 and 2 contain only half of the terms, a further assumption can be made that an average of I and II would give the best value for $\Delta P'$. The average gives us

$$\Delta P' = 1/2 E_0 I_0 \left\{ \left[(1 - \epsilon) \left(\frac{I_v}{I_0} \right)^2 \right] - 1 + \left[\frac{1}{(1 - \epsilon)} \left(\frac{E_v}{E_0} \right)^2 - 1 \right] \right\} \quad (13)$$

The wind speed can be measured by Eq. (3) of Appendix 5. The value of $\Delta P'$ from Eq. (13) can be used with the wind speed, v , in Eq. (4) to determine the value of B for a given anemometer.

Calibrating the Hot Wire Anemometer:

Once the calibration has begun, the experimenter must be careful not to put too much current through the wire since this will cause it to burn out. If this happens, the results obtained to that point will be invalid, since a new wire might possibly have different characteristics and not give the same value for B . In order to prevent wire burn-out, always decrease the power to the wire before reducing wind tunnel fan speed. See Appendix 5 for operation of the Flow Corporation Micromanometer and Appendix 4 for operation of the WT4 wind tunnel.

With the wind tunnel fan off, adjust the potential across the wire element until it reaches a dull red glow. Record the values of E_0 and I_0 . Start the wind tunnel at its slowest speed. Carefully note the voltage going to the fan motor as it will be necessary to duplicate this voltage several times. Adjust the potential across the wire element until the same dull glow is observed, and adjust the EMF until Eq. (5) is satisfied. Record values of E_v and I_v . Make a reading for each of the settings starting at 30 or 40 volts and increases of volt intervals until 120 volts is reached. Turn the power source of the hot wire to zero and shut the fan off; then start over with the $v=0$ setting. Repeat until a minimum of 10 readings are made at each setting. The wind speed v is obtained by reading the micromanometer for each of the settings of the variac control in order to complete the calibration.

In order to place the anemometer in practical use, the value of B must be known and $\Delta P'$ calculated from (13) in order to calculate v from (4a). Needless to say, a calibration chart with v vs $E_v I_v$ will be helpful in this part of the experiment. Compare the values of v from the micromanometer to those obtained by (4a) and draw your conclusions from this data. Also compare the measured values of v vs fan voltage with the values from Fig. 2. Would the value of v_{4a} be more accurate if B were a multivalued function?

See References: Betchov (1957), Flow Corporation (1958), Gill (1952, 1954), Hastings-Raydist, Inc. (1957), MacCready (1953), Middleton and Spilhaus (1953), Ower (1949), Sanford (1951), Swinbank (1951).

Laboratory Exercise #4: Calibrating Cup Anemometers

Cup Anemometers:

The cup anemometer is based on the principle that the rate of rotation of the wind propelled cups is a function of the wind speed

$$n = f(V_w)$$

where n is the number of rotations in a measured period of time and V_w is the actual wind speed. Since the anemometer is to be used to measure wind speed, we want to find the speed as a function of cup rotation

$$V_w = f(n) .$$

In 1920, Brazier suggested that the relationship was polynomial with relation to the number of revolution

$$V_w = a + bn + cn^2 + \dots \quad (1)$$

Middleton and Spilhaus stated that the Brazier equation made comparison of different instruments difficult since the ratio of the wind speed to the linear speed of the cups was not a dimensionless quantity. In fact, with the Brazier model, the linear speed of the cups is an unknown quantity and the Brazier ratio is

$$R_b = \frac{V_w}{n} . \quad (2)$$

Lest they be accused of not offering constructive criticism, Middleton and Spilhaus proposed that V_w was a function of the linear speed of the cups, v

$$V_w = a + bv + cv^2 + \dots , \quad (3)$$

and the ratio for this model is

$$R_m = \frac{V_w}{v} \quad (4)$$

It will be noted that the ratio, R_m , is a dimensionless quantity. Spilhaus later suggested that the relationship was a hyperbolic function and concluded that

$$k = \left(\frac{V_w}{v} - h \right) \left(\frac{V_w}{V_o} - 1 \right) \quad (5)$$

where v is the linear velocity of the cups, V_w is the wind speed, V_o is the speed of the air at which the cups initiate motion, and h and k are constants to be determined for an individual anemometer. The Spilhaus ratio is

$$R_s = \frac{k}{\frac{V_w}{V_o} - 1} + h \quad (6)$$

Operation of the Revolution Counting Anemometer:

The standard airways anemometer makes one contact for every twelve revolutions of the cups; however, for experimental purposes, we are using an anemometer and counter that make contact every time the cups make one revolution. It is necessary to insure that the anemometer is centered in the tunnel and that the lead wire is fastened securely. The counter can be operated by turning the switch to the ON position while simultaneously starting a stopwatch. The number of revolutions and the time are measured and from these data, both n and v can be calculated. It is suggested that a minimum of 200 revolutions be used in these calculations. Since there is no means provided for resetting the counter to zero, it may be necessary to record starting and ending counts unless the operator can quickly turn the counter off on the 200th revolution. This lead should be disconnected from either the anemometer or the counter box at the completion of the experiment.

Procedure:

In order to obtain data for this experiment, the specification of the cup anemometers must be known.

	<u>Cup Diameter</u>	<u>Radius of Inscribed Circle</u>
Airways (silver)	2.750 inches	4.75 inches
Casella (black)	2.125 inches	3.00 inches

The wind tunnel and micromanometer are operated according to the procedures in Appendices 4 and 5, respectively. The wind tunnel screens will be used to control stream velocity and the louvered panel will be used to determine the value of V_0 for the hyperbolic model. Determine whether the series or hyperbolic model is best, and which of the series models is best. Draw calibration curves for all three models, plotting V_w vs n , v , h , k on one graph. After the constants have been determined experimentally, perform an error analysis on the results comparing V_w to V_m , where V_m is the wind speed as measured by the anemometers. As an added section, measure the wind speed with the louvered panel full open. Calculate the number of revolutions per unit time that is required for this velocity. Then close the panel, allow the cups to slow or stop, and then open the louvers to their original position and measure the time it takes the anemometer to come to the correct speed. This experiment will demonstrate the problems arising when an attempt is made to measure gusty winds.

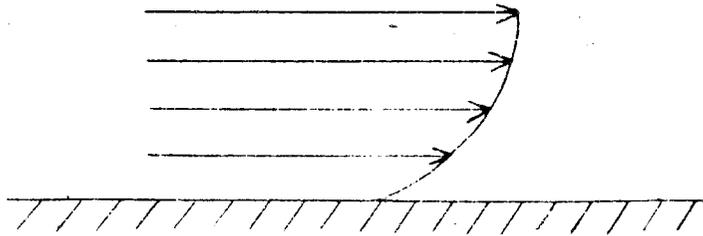
See References: Handbook of Meteorological Instruments (1958), Middleton and Spilhaus (1953), Ower (1949).

Laboratory Exercise #5: Wind Profile

If one were to consider a viscous fluid moving past a stationary plane surface, the flow being everywhere parallel to the surface, then the viscous flow near the solid boundary will exhibit a velocity profile in which the fluid velocity increases as the distance from the boundary increases. Experimental evidence suggests that the fluid particles in immediate contact with the surface are at rest. The faster moving fluid particles away from the surface tend to accelerate the slower layers nearer the surface, while they themselves tend to be retarded. Thus, there is a tangential force on the surface in the direction of motion and a velocity gradient in the fluid normal to the surface. The rate at which the velocity increases as the distance from the surface increases is represented by du/dz . In natural fluids such as air, the tangential force F acting on a unit area of the surface is proportional to the velocity gradient.

$$F = \mu \, du/dz \tag{1}$$

where μ is a constant at any given temperature and pressure, and is referred to as the coefficient of viscosity. The velocity profile resulting from the above considerations is represented in the following figure.



Viscosity flow near a solid boundary

Laboratory investigations indicate that with increase in wind speed over a given surface or as the surface becomes increasingly rough at constant wind speed, a stage is reached at which the purely viscous stress at the surface is outweighed by the effect of pressure forces associated with the roughness elements. This flow is said to be aerodynamically rough and the value of the kinematic viscosity has less of an influence on the profile. The aerodynamical roughness of the flow and its influence on the profile have been found by experiment to depend on a length which is characteristic of the surface roughness. The wind profile can be described by Eq. (2)

$$\frac{\partial u}{\partial z} = \frac{u_*}{kz} \quad (2)$$

$$\frac{u}{u_*} = \frac{1}{k} \ln \frac{z}{z_0} \quad (3)$$

where k is von Karman's constant $k = 0.4$, and the friction velocity is defined as u_* .

The friction velocity can be obtained from a solution of Eq. (3) for an adiabatic atmosphere. Kao (1959) suggested a relationship by which the friction velocity could be determined for a nonadiabatic atmosphere. This relationship is given in Eq. (4)

$$u_* = \frac{(U_2 - U_1)(z_3 - z_2) - (U_3 - U_2)(z_2 - z_1)}{(z_3 - z_2) \ln(z_2/z_1) - (z_2 - z_1) \ln(z_3/z_2)} \quad (4)$$

The laboratory portion of this experiment will be conducted as follows:

Step 1:

Using the wind tunnel and the pitot tube anemometer, determine the vertical wind profile of the wind tunnel, extending the pitot tube 2 inches above the lower interior surface of the wind tunnel.

Insert screen #1, turn on the wind tunnel and read the manometer. Then repeat, using screen #14. Rotate each of the screens until 10 readings have been made.

Step 2:

Then raise the pitot tube to 4 inches above the bottom and repeat the procedure of measurements.

Step 3:

Raise the pitot tube to 8 inches above the bottom of the wind tunnel and repeat the procedure of measurements.

Step 4:

Compute wind profiles for the interior of the wind tunnel which show the variations of wind speed from the bottom of the wind tunnel and include the accuracies of each for the mean wind speed values at each of the 2, 4, and 8 inch measurements. Also determine and compare the friction velocities obtained from Eqs. (3) and (4). Discuss the results of this experiment and include in the write-up a plotted wind profile of the values that you obtained.

See References: Kao (1959), Ower (1949), Swinbank (1951).

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24. Swinbank, W.C., 1951: The Measurement of Vertical Transfer of Heat and Water Vapor. J. Meteorol., 8.
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4.4 PRESSURE

4.4.1 PREREQUISITES

University Physics and concurrent enrollment in Differential Equations.

4.4.2 THEORETICAL CONSIDERATIONS AND RATIONALE

In order to understand the operation of pressure sensors, it is necessary to know what is meant by the word "pressure." We shall assume pressure to be the force applied to, or distributed over, a surface, measured as a force per unit area. Another consideration which is useful is Pascal's Law--"A hydrostatic principle that pressure applied to an enclosed fluid is transmitted undiminished to every portion of the fluid and to the walls of the containing vessel."

This section is designed to assist the student to obtain a better understanding of atmospheric pressure and to provide a "HANDS ON" experience of measuring atmospheric pressures using Pascal's Law and Clapeyron equation, which is the differential equation relating pressure to temperature in a system in which two phases of a substance are in equilibrium.

Definitions

Pressure: Pressure is a force per unit area exerted on a surface by a fluid, liquid, or gas. Pressure at any point in a fluid is exerted equally in all directions. The dimensions are $ML^{-1}T^{-2}$. Since pressure is exerted equally in all directions, it can be thought of as energy per unit volume. Since it is more often thought of as a force per unit area, we shall use that term in our discussion. The units of pressure were agreed upon by the General Conference on Weights and Measures, which met in Paris, France in October 1960, and gave the name Systeme International des Unites (SI) to the metric system based on the meter as the unit of length, the kilogram as the unit of mass, the second as the unit of time, the ampere as the unit of electric current, the Kelvin as the degree of temperature, and the candela as the unit of luminous (light) intensity. The SI system has a single unit for pressure called the pascal (Pa), which is defined as the pressure resulting when a force of 1 newton acts uniformly over an area of 1 square meter. Thus $1 \text{ Pa} = 1 \text{ N/m}^2$.

Another unit of pressure often used for high pressure work is the bar, which is defined as one million dynes per square centimeter. This was suggested by V. Bjerkness, the famous Norwegian hydro-dynamist. Because it was awkward for most atmospheric pressures to be in fractional parts, the millibar was used so that atmospheric pressures could be given in whole units. The millibar can be converted into pascals by multiplying by 100 or $1 \text{ Pa} = 10^{-2} \text{ mb}$. In the

discussion of pressure, meteorologists use the units of millibars. Occasionally, when discussing altimeter settings, etc., inches or millimeters of mercury will creep into the conversation.

The pressure gradient is defined in the Glossary of Meteorology, (Huschke, Ed., 1959) as "the rate of decrease (gradient) of pressure in space at a fixed time." A pressure gradient exerts a force which acts on the air by virtue of the variation of pressure in space at a fixed time. The force is normal to the surfaces of constant pressure; the sign is negative because the force acts from high to low pressure.

In meteorological dynamics, the pressure gradient force acting per unit mass of air is a significant force and is normally referred to as the "pressure gradient force." The balance in the atmosphere between the horizontal pressure gradient force and the horizontal Coriolis forces results in geostrophic wind.

General Principles

Much of the following material is based on information from the following: Meteorological Office (1981); Middleton and Spilhaus (1953); Klein (1974); Lion (1959); Middleton (1969). The measurement of the weight of air was first realized by Galileo when he performed an unusual experiment to determine whether air possessed weight. Using some type of a pressure pump, he squeezed into a great glass flask as much extra air as he could, sealed it, and weighed it. (If this experiment were performed today, it would be said that the glass flask was pressurized.) Galileo weighed the flask and the additional air which equaled A units of weight. He then allowed the excessive air pressure to escape, reweighed the flask, and found the weight B. He reasoned that the difference given by A minus B corresponded to the weight of the additional air that was released. Most of you have experienced weighing excessive air when you change a flat tire.

Torricelli worked with Galileo in 1642 during the last few months of his life. This time Galileo must have greatly inspired Torricelli, as he is generally credited with inventing the mercurial barometer in 1643.

Mercury Barometers

The mercurial barometer measures the pressure exerted by the atmosphere by weighing a column of the atmosphere. This was first done in an experiment performed by Torricelli in 1643. It should be noted that Torricelli was attempting to prove Aristotle in error because of his idea that the existence of a vacuum anywhere in the universe was impossible. Aristotle believed the universe was of finite size, without admitting the existence of an empty space around its external boundary. Middleton (1969) also reports that Aristotle claimed light could not penetrate a vacuum. In 1643 in Italy, Torricelli attempted to create a vacuum. Torricelli used a long tube,

which was sealed at one end, and a bowl of quicksilver, part of which was used to fill the tube. He then placed his finger over the open end of the tube filled with quicksilver and inverted the tube, immersing the end covered by his finger into the quicksilver. Then he removed his finger from the bowl of quicksilver, while holding the tube in a vertical position with the other hand. Torricelli then attached a cord to the closed end and attached the tube to the wall with the open end submerged in the quicksilver reservoir. He then claimed that the clear space in the upper end of the tube was a vacuum, which in fact it was. Before leaving the laboratory and retiring for the evening, he made a mark on the wall to denote the upper level of the quicksilver. It is reported that he marked the wall each morning and evening denoting the date, time, and height of the column. He noted that these marks were at different heights on each of the various days. This caused him to ask why the height of the quicksilver varied with time.

He reasoned that we live at the bottom of an ocean of air, and that this ocean has tides as do oceans of water. The thickness of this ocean of air, he speculated, changes with the movements of the tides.

Torricelli then applied Eq. (5) and assumed that the density of the air was constant and that the pressure would decrease as the height increased as is demonstrated in Eq. (7). Similar experiments were performed by others prior to Torricelli, but Torricelli is credited with the invention of the barometer because he recognized that a vacuum did not possess an adhesive material, which held the quicksilver in the tube, but that the weight of the atmosphere prevented the quicksilver from flowing out of the tube. According to Middleton, this information left the Italian peninsula by way of a letter written to Marin Mersenne by a fellow Frenchman living in Rome at the time. Pascal heard of the Torricelli experiment and suggested that his brother-in-law, Florin Perier at Clermont, repeat the Italian's experiment and determine if the pressure of the air would decrease as one increased elevation, as Torricelli suggested. In 1648 on September 19, Perier and five friends took the Torricellian tube up the Put-de-Dome. Perier left a second tube as a control at the monastery in Clermont to be watched and observed by a monk. The monastery tube read 26 inches, 3 1/2 lines and remained at this height all day. In the tube which Perier and his five friends took to the top of the mountain, the height of the quicksilver was measured several times and read 23 inches and two lines. Perier compared the tubes upon returning to the monastery after his return in the afternoon. These results were published in the fall of 1648.

Mercury barometers can be divided into two basic subdivisions: barometers with cisterns, and barometers without cisterns (siphon barometers). The cistern barometers can also be divided into two

subclasses. These are those with adjustable cisterns, the most notable example of this subclass is the Fortin barometer, and those with fixed cisterns. The most popular example of this is the marine or Kew barometer.

The physical principle on which the cistern barometers function is that of weighing or balancing. These barometers balance the weight of a column of air reaching from the surface to the top of the atmosphere with a column of mercury. At sea level, the column of mercury extends 760 mm. This physical relationship can be demonstrated as a mathematical expression of a force. The force exerted by the column of mercury can be written as

$$F = \rho ghA , \quad (1)$$

where ρ is the density of the mercury, g is the acceleration of gravity, h is the height of the column of mercury, and A is the area of the surface of the column. With this configuration, the weight of the column of air rests on the surface area of the mercury in the cistern. The two forces exerted by the mercury column and the atmosphere are balanced one against the other. Since these forces are acting against the surface of the other, they exert pressure on those surfaces in contact with them. If we recall that pressure is defined as force per area, then we readily observe that by dividing the area term A into both sides of Eq. 1 the pressure is obtained and we have

$$P = F/A = \rho gh , \quad (2)$$

where P is the pressure exerted by the mercury column, and if this is balanced against the atmosphere, then it is also representative of the atmosphere at that location. Now if we do as Torricelli did and determine the density of the air at approximately sea level and solve for h , we obtain an atmospheric height of about 8 km. This is assuming that the atmosphere has a constant density, which is incorrect since the density of the atmosphere decreases with the height. While we don't know how high the real atmosphere reaches, we do know the pressure value of the column of mercury. Since the column of mercury is balanced against the column of air, we shall use the pressure value measured by the mercury column as the air pressure. This is correct so far, but we must make allowances for the thermal expansion of the mercury (after all that particular property of mercury is utilized in metal and glass thermometers). We also must make corrections for the variation of gravity for both elevation and latitude. This is done because gravity is defined at 45° N average sea-level. The gravity correction is obtained from Eq. 3 as

$$g_{\phi h} = g_{45, 0} (1 - 0.000259 \cos 2\phi) (1 - 0.000,000,06Z) \quad (Z \text{ in m}).$$

Next we must correct for the amount of thermal expansion which has occurred in the mercury column.- This is done by using the temperature correction table from the National Weather Service. This correction, the gravity correction, and the instrument correction are algebraically added to the raw reading. The results of this procedure give the correct station pressure. The corrected station pressure provides the basis for determining the value of the sea-level pressure and the altimeter setting for the station. In order to compute the sea-level pressure, it is necessary to approximate the lapse rate between your station and average sea-level. What is the lapse rate the atmosphere would have if it were not for the mud, dirt, rocks and solid granite between your elevation and sea level? An approximation for this has been statistically determined by use of the mean 12-h temperature, and for stations whose elevation is greater than 2500 ft, a plateau correction.

The foregoing was a description for determining the pressure, using an adjustable cistern barometer after the cistern level was raised to intersect the tip of the ivory fiducial point. This procedure is followed in making pressure readings with a Fortin type mercurial barometer.

The fixed-cistern mercurial barometers present one additional correction which is built into the barometer scale. Since the law of conservation of mass must be obeyed, one can easily recognize that only a given amount of mercury is available in the tube and cistern of a barometer. Thus, when the mercury in a fixed-cistern barometer rises in the tube, its level must be lowered in the cistern to steady the sloshing of the mercury. The Kew barometer is mounted on the gimbals to allow it to hang vertically on board a rolling ship. The advantage of the Kew fixed-cistern barometer is that it is not necessary to adjust the mercury level in the cistern as is done in the Fortin barometer. The disadvantage is that the observer does not know the correct distance between the mercury level in the cistern and its level in the tube. To overcome this shortcoming, the cistern is constructed with a surface area (A) with respect to the area of the tube (S). This means that a correction can be made for this uncertainty and is performed in the following manner. The length of the marked scale on the tube is reduced to the following ratio to correct for this error produced. Use the following formula for scale reductions to correct for the apparent violation of the law of conservation,

$$\text{scale unit correction} = \frac{A}{S + A} \quad . \quad (3)$$

The manufacturer constructs these barometers so that the scale unit correction is between 0.95 and 0.99. This greatly reduces the problem.

Siphon barometers are without cisterns and have the general shape of manometers in that only one end of the tube is sealed and the other end is open. This barometer is often in the shape of a U tube. The mercury level is measured in the following manner: The mercury in the closed side rests much higher than that in the open end. This is shown in Fig. 4-19.



Fig. 4-19. Principle of a Siphon Barometer.

The mercury in the closed side of the siphon barometer is resting much higher than is the mercury on the open side in the figure. The force acting on side C is

$$F_c = \rho_{hg} g h_{ac} A \quad (4)$$

and the force acting on side O is

$$F_o = \rho_{hg} g h_{bc} A + p_a g h_{bt} A \quad (5)$$

(where t refers to the top of the atmosphere).

Equating and dividing by the area of the mercury surface area A, we obtain

$$P_c = P_o \quad (6)$$

and from the relationship in Eq. (5), we can obtain the atmospheric pressure as

$$P_a = \rho_a g h_{bt} \quad (7)$$

$$P_{bc} = \rho_{hg} g h_{bc} \quad (8)$$

$$P_o = P_a + P_{bc} = P_c \quad (9)$$

Therefore,
$$P_a = P_c - P_{bc} \quad (10)$$

where P_a is the pressure exerted by the atmosphere from the mercury of the open end of the tube to the top of the atmosphere, P_{bc} is the fraction of the pressure exerted by the mercury column to the level of the bottom of the U tube, and P_c is the total pressure or sum of $P_a + P_{bc}$.

Siphon Barometers

Siphon barometers are of two general classes. The first has a scale fixed to the tube, and the second has an adjustable scale, which can be adjusted to the top of the mercury surface in the open end of the tube. Thus the difference between the mercury surface in the open and closed ends of the tube can be read directly.

Elastic Barometers

Middleton and Spilhaus (1953) describe these barometers as a solid mechanical system in which the elastic deformation of a solid system is used as an indicator of atmospheric pressure. These barometers are subdivided into two types: Bourdon barometers and Vidie barometers, which is also called the aneroid barometer in English speaking countries (the Bourdon barometer is also "aneroid," meaning without liquid).

The Bourdon barometer is only in minor use at the present time. It consists of a closed, curved tube of elliptical cross section, which reacts to changes in the barometric pressure. The pressure changes result in the radius of curvature of the tube, one end being fixed and the motion of the other end being magnified through a system of levers attached to a pointer, which indicates pressure. These pressure gauges have little meteorological use as they are used primarily as pressure gauges for steam boilers.

The Vidie barometer, the classical aneroid barometer, has two parts. First is a closed chamber of a thin pliable metal, partly or fully evacuated with a strong spring. The spring is used to keep the box from collapsing under high atmospheric pressure. It operates on the same principle of weighing or balancing as does the mercurial barometer. It differs from the mercurial barometer in that instead of balancing the atmosphere against a column of mercury, it balances the force of the atmosphere against the combined forces of the spring and the residual air pressure in the aneroid cell. The metal chamber is thin and flexible, meaning the elastic properties of the spring determine those of the whole instrument.

The second part is an external spring. The connection between the aneroid chamber and the spring is made by means of a knife-edge, which passes through the postattached to the two sides of the chamber. These posts pass through rectangular slots in the spring. Changes in the thickness of the chamber, brought about by changes in the atmospheric pressure, are magnified by a system of levers and indicated on a scale. This transducer can provide either an electrical or graphical output. It is the change in deflection of the spring as the load, due to the variation of the atmospheric pressure on the chamber, is measured. The aneroid cells can be connected in series, which mechanically amplifies the sensitivity of the sensor. This amplification of the sensitivity is used in aircraft altimeters and other pressure sensitive instruments.

The aneroid has certain inherent advantages over mercury barometers, such as absence of liquid, portability, low weight, and easy adaptation to record data. Usually, though, they are less accurate than a mercurial barometer. They can be made very precise as is indicated in aircraft altimeters, but they need to be periodically compared to a mercurial barometer in order to maintain calibration.

Barometer Errors

Mercurial barometers have three corrections which must be applied to their readings in order to filter out inaccuracy in the data. These are (1) correction of index error, an instrument error which is caused by a nonuniform tube and is listed with the barometer from the factory calibration information (also included in this category are errors due to capillarity); (2) correction for standard gravity; (3) correction for temperature.

The General Errors of Measurement--the effect of wind-air motion produces either a positive or negative pressure variation, and it is not necessary for the barometer to be exposed directly to the wind

because dynamic fluctuations can be superimposed on the static pressure due to gustiness of the wind. Strong winds can result in barometric fluctuations of as much as 3 mb. This pumping action of the barometer is not only a function of the wind speed but also depends on the configuration of winds, doors, chimneys, and other openings in the room where the barometer is hung.

The temperature correction also introduces uncertainties, but these are minimized by keeping the barometer in a room maintained at room temperature. One assumption that is made is that the attached thermometer is at the same temperature as the mercury column of the barometer or the metals of the aneroid barometer.

Aneroid barometers have the same sources of error as mercury barometers except for the errors due to gravity since aneroids are not weighing devices. One of the more interesting errors is hysteresis due to the elasticity of the metal of the aneroid. An investigation of hysteresis using aneroids found both long- and short-term effects. This error varies with temperature, range of pressure, and rate of pressure change. Hysteresis can manifest itself in the following way. Consider an aneroid in equilibrium at a pressure of 1000 mb reduced to 500 mb.

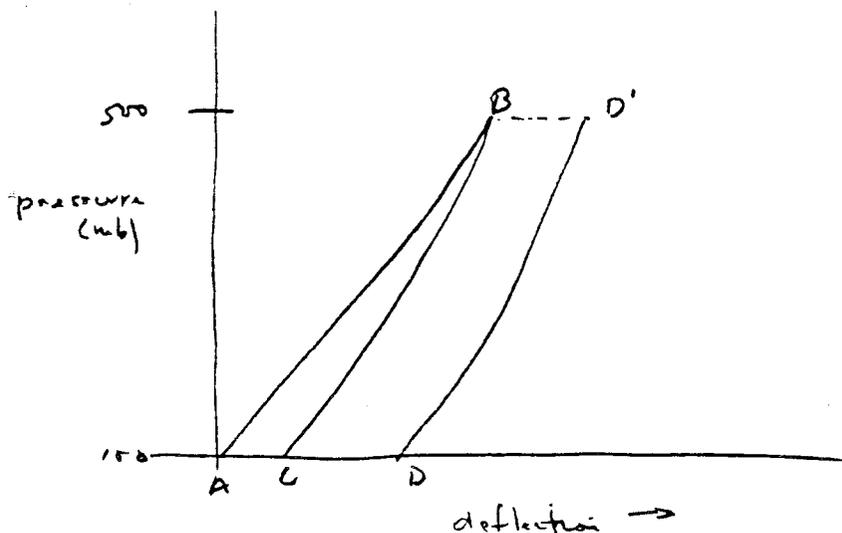


Fig. 4-20. Illustration of hysteresis.

Figure 4-20 shows the path A-B along which the deflection changes. If the pressure is then increased to 1000 mb, the deflection will be along path B-C. Subsequent cycle will result in steady-state curve D-D. Then with the passage of time and the pressure kept at 1000 mb, the deflection will return from D to A.

In practice this presents little problem because of the limited range and the small rate of change of pressure involved with surface measurements. In the case of the radiosonde, the factory calibration of the aneroid is made under the conditions of decreasing pressure only, as in its application.

Hypsometers

The ordinary hypsometer measures the boiling temperature of a liquid at which the vapor pressure of the liquid equals the pressure above its surface. The relation between the vapor pressure and temperature is known by the Clapeyron Equation (Eq. 11), which measures the boiling point with an instrument known as the hypsometer--an instrument used to determine the atmospheric pressure.

$$\frac{d(\ln P)}{dT} = \frac{L}{RT^2} \quad , \quad (11)$$

where L is the latent heat of vaporation, which for any liquid is a function of the temperature alone, and R is the gas constant, it must be recalled that the temperature is in Kelvins. The sensitivity of the hypsometer increases as the pressure decreases. This is evident when examining Eq. 11. Since the sensitivity of the instrument decreases at high atmospheric pressures, this results in measurements of poor accuracy at normal atmospheric pressures. The hypsometer is used as an auxiliary pressure sensor on radiosondes for the lower pressures, upper regions of the atmosphere.

Exposure

The barometer should be installed with the exact elevation of its fiducial point known and its latitude and longitude known to facilitate its location. These data will permit the gravitational corrections to be made. The barometer should be located in a room of a permanent structure. It is best to locate the barometer on an interior wall where it will not be subject to direct light and in a well ventilated position in the room. The air in the room should not be subject to stratification, vibrations, or rapid variations in temperature. Special efforts should be made to minimize dynamic pressure variations in that a representative measurement of the static atmospheric pressure be made. These procedures are recommended by the Meteorological Office (1981), Middleton and Spilhaus (1953).

Instrument Selection and Recording

The instruments under formal consideration in this section are the Fortin mercurial barometer and the precision aneroid barometer and microbarograph. If one were interested in accurate measurements of pressure at a fixed location, the Fortin mercurial barometer is the better choice. If several locations are being considered, the choice would be the microbarograph or precision aneroid because of its portability. The mercury type barometer is more difficult to transport quickly from one site to another and simultaneous measurements could be costly because of manpower requirements.

A microbarograph is a convenient instrument to use because it provides a continuous record. If accurate absolute measurements are required, frequent calibrations or comparisons with a Fortin barometer should be made. When pressure tendencies are needed, the microbarograph is an appropriate instrument.

If many measurements are required over the span of an hour, for instance, the microbarograph or precision aneroid is the instrument to use. If it is necessary to increase the sensitivity, this can be done by increasing the rotation of the recording drum. One must always recall that the limiting factor of this system is its time constant.

Kelleher (personal communications, 1983) has used fast recording microbarographs to study the pressure fields associated with thunderstorms in Oklahoma. There are other instruments which may be of use. If one wishes to examine the small pressure fluctuations, which would go unnoticed on the average microbarograph record, the instruments to use are the variometers or the variographs. There are two basic types of these instruments, and they are classified as whether they record pressure or the time differential of pressure (dp/dt). The variograph, which measures pressure, is an aneroid with optical magnification of its output. The magnification is usually limited by the internal friction of the system.

The second type of variograph is for measuring dp/dt . One of the difficulties of this instrument is the zero adjustment at frequent intervals. This sensor utilizes a large closed chamber with a small air leak and a technique for measuring the difference between the internal and external pressures of the chamber. The air leak is made with a piece of capillary tubing. For more details see Middleton and Spilhaus (1953).

The use of the barometers and microbarographs results in analog and digital records. Both of these outputs can be recorded digitally with a P.C. recorder (Pike and Bargaen, 1969). The NCAR Portable Automated Mesonet (PAM II) transducer utilizes capacitance as the pressure changes. Since the capacitor is part of an oscillator circuit, its frequency of oscillation is

$$f = \frac{1}{kRC} ,$$

where R is the resistance, k is a constant, and C is the capacitance which varies as the pressure varies in the design of the aneroid used in the mesonet. Thus the frequency is measured with a microprocessor which converts this frequency to a pressure.

Other transducers that lend themselves to electronic recording are crystal resonance gauges, strain gauges, and variable reluctance gauges. These are described briefly by Dobson, Hasse, and Davis (1980).

NCAR also uses two other pressure sensing devices as calibration mechanisms. These are a piston type secondary pressure standard and a Hamilton "vibrating cylinder," which serves as a pressure transfer standard. These two instruments make up the pressure calibration facility at NCAR.

4.4.3 LABORATORY EXERCISES

Laboratory Exercise #1: Measurement of Atmospheric Pressure

Objective:

The objective of this lab is to have you gain experience using standard instruments to measure atmospheric pressure at station elevation in inches of Hg and mb. The instruments will be a Fortin barometer, a microbarograph and an aneroid barometer.

Method:

Read the Fortin barometer as follows:

- Open the case and read and record the instrument temperature.
- Turn the light switch on and use the adjustment screw to raise the level of the mercury in the cistern so that it just touches the fiducial point.
- Using the vernier, record the height of the mercury column to the nearest 0.002" Hg. Let this quantity be B_0 .
- Lower the mercury level in the cistern below the fiducial point, turn off the barometer light, and close the case.

Read the microbarograph located just above the Fortin barometer. Call this reading B_m .

Read the aneroid barometer. Call this reading B_a .

Make corrections to B_0 . (There are three corrections that should be made to B_0 : instrument, local gravity, and temperature of mercury. Only the latter two will be performed and both serve to transform the reading to standard conditions so that mercury column height can be easily converted to mb.)

- Temperature of mercury: Use the table posted near the barometer. This difficulty in preparing such a table can be sensed by reading about this correction in the Smithsonian Meteorological Tables. Let B_0 corrected for temperature be called B_t .
- Local gravity: The reference for the following discussion is that cited above. The appropriate formula is

$$C = \frac{g_\lambda - g_0}{g_0} B_t \quad (1)$$

where C = correction to be applied to B_t

B_t = height of Hg column, corrected for temperature and instrument errors

$$g_0 = 980.665 \text{ cm s}^{-2}$$

g_λ = local acceleration due to gravity (cm s^{-2}) .

Acceleration due to gravity at sea level at latitude ϕ :

$$g_\phi = 980.6160(1 - 0.0026373\cos^2\phi + 0.0000059\cos^2 2\phi) \text{ cm s}^{-2} .$$

Acceleration due to gravity at elevation $z(\text{m})$ away from sea level:

$$g_\lambda = g_\phi - 0.0003086z.$$

The equation for acceleration due to gravity at sea level at latitude ϕ says that each time a pressure observation is made the correction will be different. However, a very good approximation to C can be made by using an average value for the height of the Hg column at a station. Call this value B' so that the formula becomes

$$C' = \frac{g_\lambda - g_0}{g_0} B' . \quad (2)$$

The value of C' is then added to B_t to get the station pressure for standard conditions. Call this quantity B_s .

Obtain the height of the Hg column for standard conditions. (Show your work.)

- Compute B' . Determine the instrument height from the Norman quadrangle topographic chart and the height of the instrument above the ground. Use the pressure-height relationship for the U.S. Standard Atmosphere to determine the associated height of the mercury column.
- Compute g_ϕ . Determine the value of g_ϕ after finding the latitude of the Engr. Lab from the topographic chart.
- Compute g_z . Given g_ϕ and z , g_z can be found.
- Compute C' .
- Compute B_s .

As you have just experienced, the computation of the gravity correction is messy and would be worse if we used C instead of C' . Thus, one calculates the gravity correction one time only.

Fill in the column for Observation 1 for the Fortin barometer and give the station pressure B_s in inches and millibars.

Obtain the barograph correction relative to the station pressure B_s , computed from the Fortin barometer, and fill in Observation 1.

Determine the aneroid correction relative to the station pressure B_s and fill in Observation 1.

Take four additional readings of the three available barometers on as many different days and times as possible. Read all the instruments at approximately the same time (within 10-15 min). Fill in Observations 2 through 5.

Additional questions:

- Are the corrections for the microbarograph and aneroid constants? If not, what are the possible reasons for this?
- The following will give you a feeling for the calculation of geostrophic winds from pressure gradient measurements. Suppose you are interested in computing the geostrophic wind speed from the difference in the station pressures between Oklahoma City (OKC) and Norman. Assume that the barometers at both places have an accuracy of +0.1 mb.

This means that an error of 0.2 mb could be introduced in the measured pressure difference between OKC and Norman. What error would this introduce in the calculation of the geostrophic wind speed (V_g) in $m\ s^{-1}$?

$$V_g = \frac{1}{2\rho\Omega\sin\phi} \frac{\Delta P}{\Delta S} ,$$

where ΔP = Pressure difference between OKC and Norman

ΔS = Distance between OKC and Norman

ρ = air density

Ω = $(\pi/12)\ h^{-1}$

ϕ = latitude.

Laboratory Exercise #2

Objective:

Learn how to use a Fortin barometer, a microbarograph, and an aneroid barometer.

Equipment:

- Fortin barometer
- microbarograph
- aneroid barometer

Laboratory Exercise #3: Pressure Calibration

Objective:

To perform a static calibration on a portable barometer.

Equipment:

- Barometer
- Piston gage reference system

NAME _____

Observation	1	2	3	4	5
DATE					
TIME					
FORTIN BAROMETER B_0					
TEMP					
TEMP CORR.					
GRAVITY CORR.					
inches B_s					
mb					
MICROBAROGRAPH B_m					
CORR.					
ANEROID B_a					
CORR.					

AVERAGE MICROBAROGRAPH CORR. _____

AVERAGE ANEROID CORR. _____

Method:

The instructor will introduce you to the operation and techniques of using the equipment. Obtain calibration points at both below and above ambient pressure.

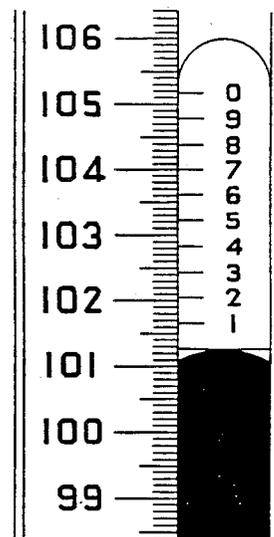
Results:

The generated pressures must be corrected for nonstandard local acceleration of gravity, for nonzero reference vacuum, and for any departure of the piston gage from its calibration temperature. Software routines are available for this. Compare the reference pressures with the barometer measurements in any suitable way.

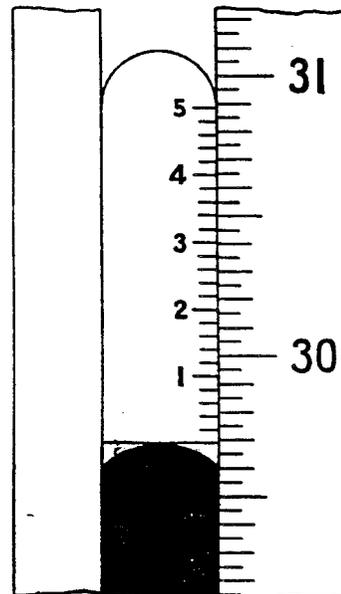
Questions:

- The manual for the piston gage calls it a "Primary Pressure Standard." Is it? Why or why not?
- The piston gage was calibrated at the National Bureau of Standards. Is the portable barometer now "traceable to NBS"? If you dropped it, would that affect its traceability? If you bumped it? How can you tell if it is still traceable? Suppose you loaned it out, what then?

ADDITIONAL FIGURES



(a) millibar graduations



(b) inch graduations

FIG. READING THE BAROMETER SCALE

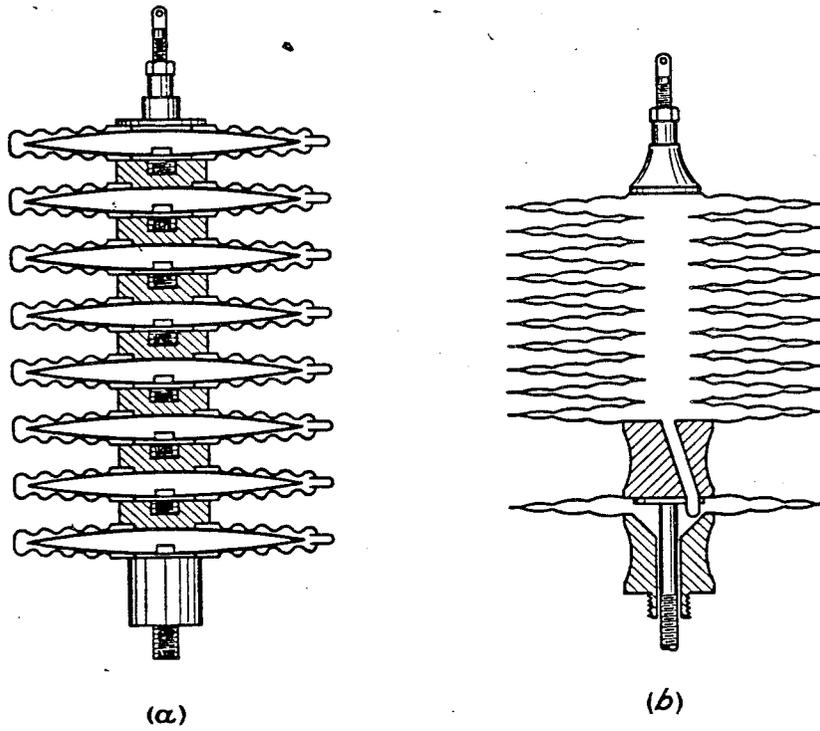


FIG. ANEROID CELLS FOR BAROGRAPHS

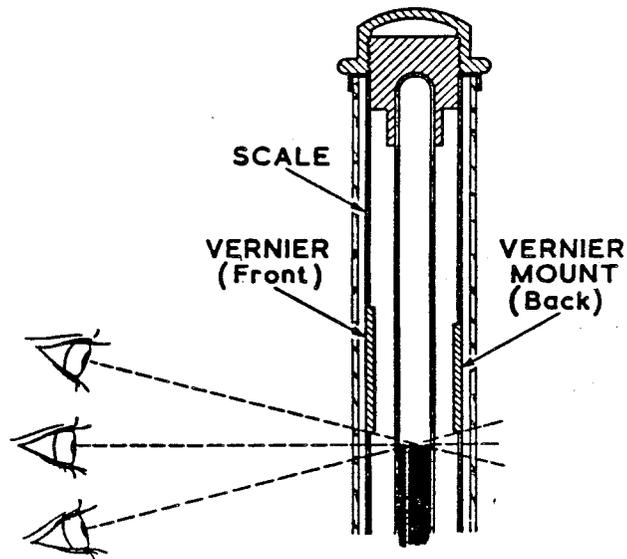


FIG. EFFECT OF INCORRECT EYE LEVEL IN READING THE BAROMETER

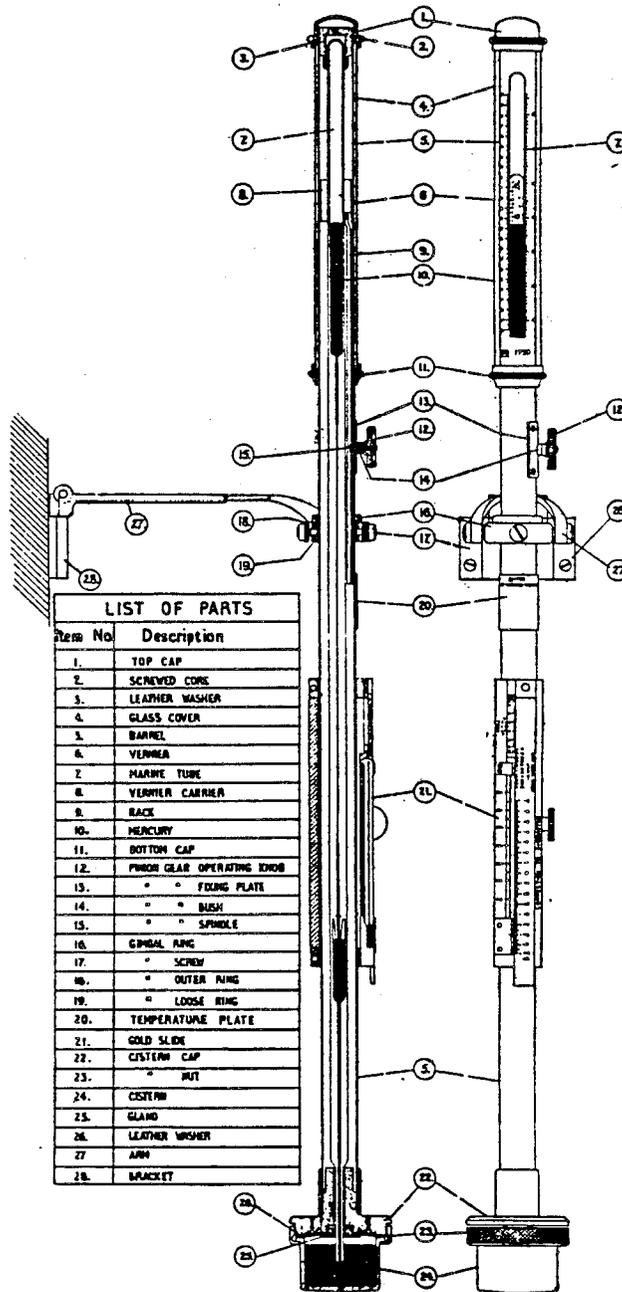


FIG. 8—METEOROLOGICAL OFFICE KEW-PATTERN
MARINE BAROMETER
with barometer-correction slide Mk IV (Gold slide)

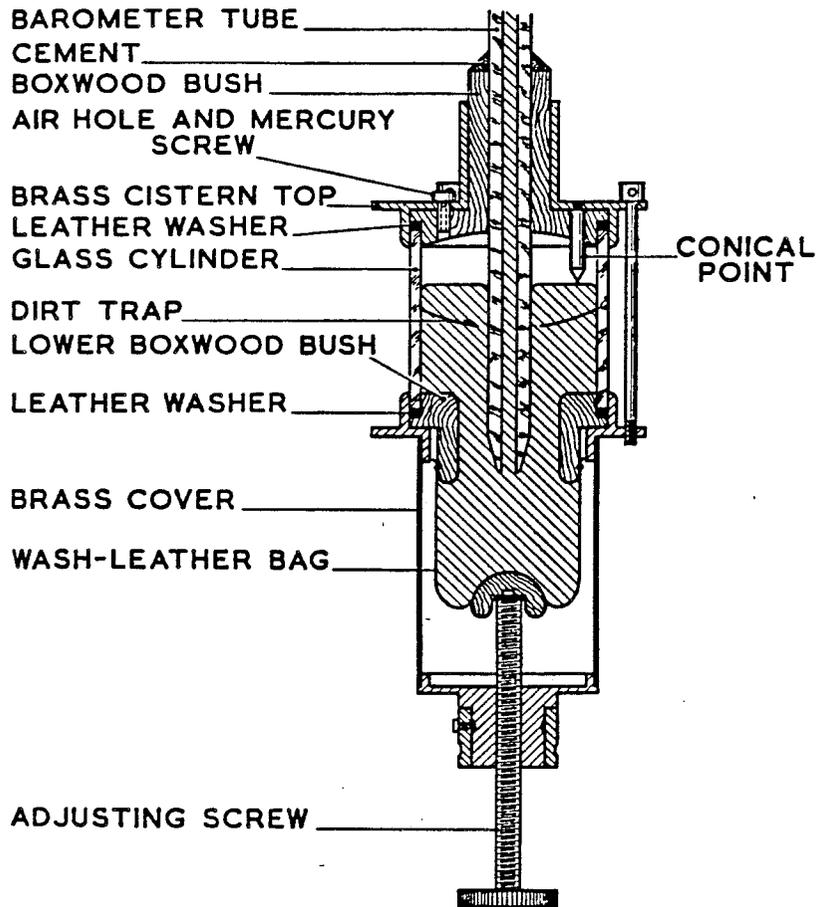
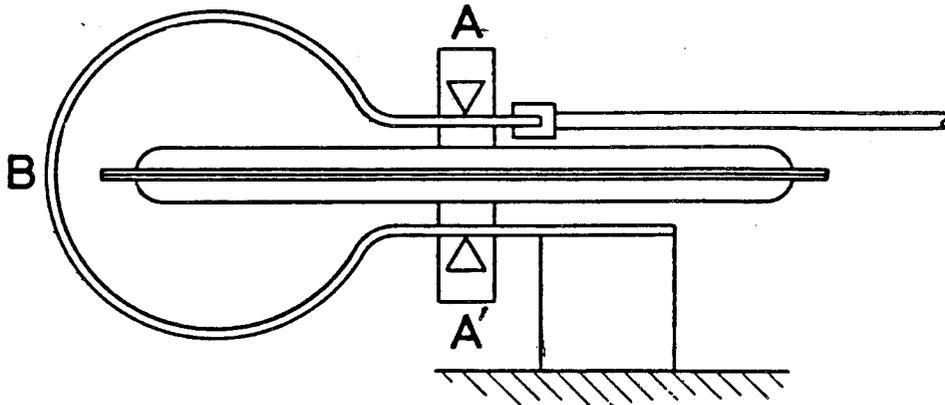


FIG. -CISTERN OF A FORTIN BAROMETER

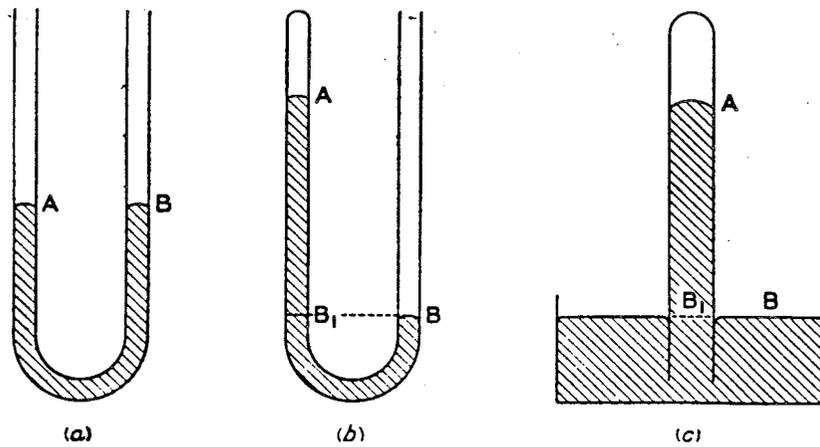


FIG STAGES IN THE DEVELOPMENT OF THE BAROMETER

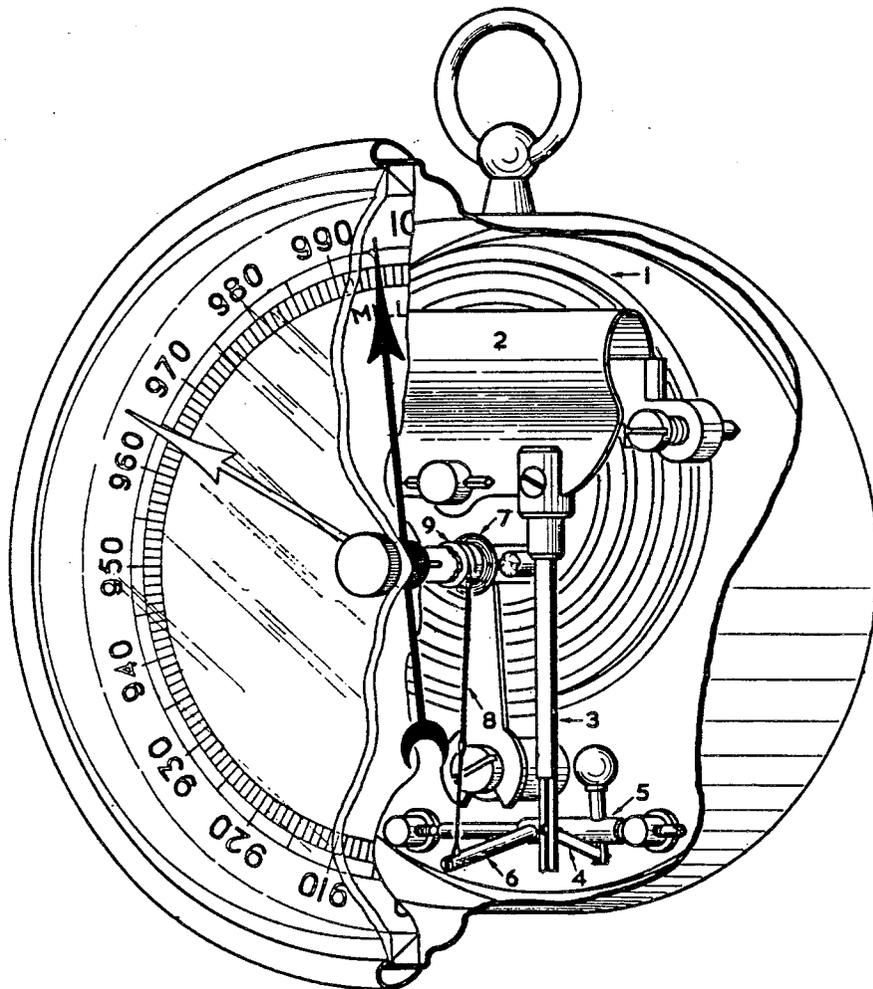


FIG. —TYPICAL MECHANISM OF AN ANEROID BAROMETER

4.5 RADIATION

4.5.1 REQUIRED BACKGROUND

In order to understand the function of various radiation instruments, it is important to have a background in the elements of electromagnetic radiation as they affect atmospheric radiation measurements. Thus, it is important to understand the spectrum of electromagnetic radiation, the positional and tilt relationships of the earth to the sun, and the global energy balance. These elementary aspects of atmospheric radiation are covered in beginning texts in meteorology such as Elements of Meteorology (Miller et al., 1983, see Chap. 4) and Atmospheric Science (Wallace and Hobbs, 1977).

Some theoretical background, culminating in the radiation laws, is also needed. The scope of this background is given in the following subsections.

Terminology and Units

The terminology for various components of radiation as given below is quite common.

Flux or flux density of radiation. Radiant energy per unit time per unit area. It refers to the radiant energy received on a plane surface or passing through a plane.

Direct solar radiation or direct radiation. The energy per unit time per unit area (flux) received on a surface from and normal to the sun's disk.

Diffuse radiation. The scattered flux of solar radiation from the sky incident on a horizontal surface (global/direct radiation).

Global radiation. The total flux of solar radiation (direct plus diffuse) incident on a horizontal surface.

Short-wave radiation. Radiation from about 0.3 to 4 μm .

Long-wave radiation. Radiation from about 4 to 50 μm .

Terrestrial radiation. Radiation originating at the earth's surface or in the atmosphere, which is all long-wave radiation.

Spectral intensity. The radiant energy per unit time per unit area per unit solid angle per unit wavelength received on a plane normal to the beam of radiation. It refers to a beam of radiant energy contained in a cone normal to a surface or plane. By appropriate integration of spectral intensity, various other components can be obtained.

The units of spectral intensity are

$$1 \text{ Wm}^{-2} \text{ster}^{-1} \text{cm}^{-1} = 1.433 \cdot 10^{-3} \text{ cal cm}^{-2} \text{min}^{-1} \text{ster}^{-1} \text{cm}^{-1}$$

$$1 \text{ cal cm}^{-2} \text{min}^{-1} \text{ster}^{-1} \text{cm}^{-1} = 698 \text{ Wm}^{-2} \text{ster}^{-1} \text{cm}^{-1}$$

If one integrates over the hemisphere (2π steradians) above a unit surface using spherical coordinates, one obtains spectral radiant flux (or flux density), for which the units are

$$1 \text{ Wm}^{-2} \text{cm}^{-1} = 1.433 \cdot 10^{-3} \text{ cal cm}^{-2} \text{min}^{-1} \text{cm}^{-1}$$

$$1 \text{ cal cm}^{-2} \text{min}^{-1} \text{cm}^{-1} = 698 \text{ Wm}^{-2} \text{cm}^{-1}$$

Finally, if one integrates over the wavelength span of interest, the units of radiant flux (or flux density) are

$$1 \text{ W cm}^{-2} = 1.433 \cdot 10^{-3} \text{ cal cm}^{-2} \text{min}^{-1}$$

$$1 \text{ cal cm}^{-2} \text{min}^{-1} = 698 \text{ Wm}^{-2}$$

The terminology given below for radiation instruments is based on the World Meteorological Organization (1965) classification.

Pyrheliometer. An instrument for measuring the intensity of direct solar radiation at normal incidence (i.e., sensor surface perpendicular to the sun's rays and with a narrow field of view). Most designs have an aperture of 5° - 6° field of view thus measuring the circumsolar radiation component in addition to the direct normal component. This robustness in field of view is to reduce solar tracking as a source of error.

Pyranometer. An instrument for the measurement of the solar radiation received from the whole hemisphere. It is suitable for the measurement of the global or diffuse (sky) radiation and may be tilted to any angle to measure reflected solar radiation as well (with a possible loss of accuracy).

Pyrgeometer. An instrument for the measurement of the net radiation on a horizontal upward facing black surface at the ambient air temperature.

Pyrradiometer. An instrument for the measurement of both solar and atmospheric or terrestrial radiation (long- and short-wave or total radiation).

Net Pyrradiometer. An instrument for the measurement of the net flux of upward and downward total (solar, terrestrial surface and atmospheric) radiation through a horizontal surface.

Examples of different radiometers are shown in Figs. 4-21, 4-22, and 4-23.

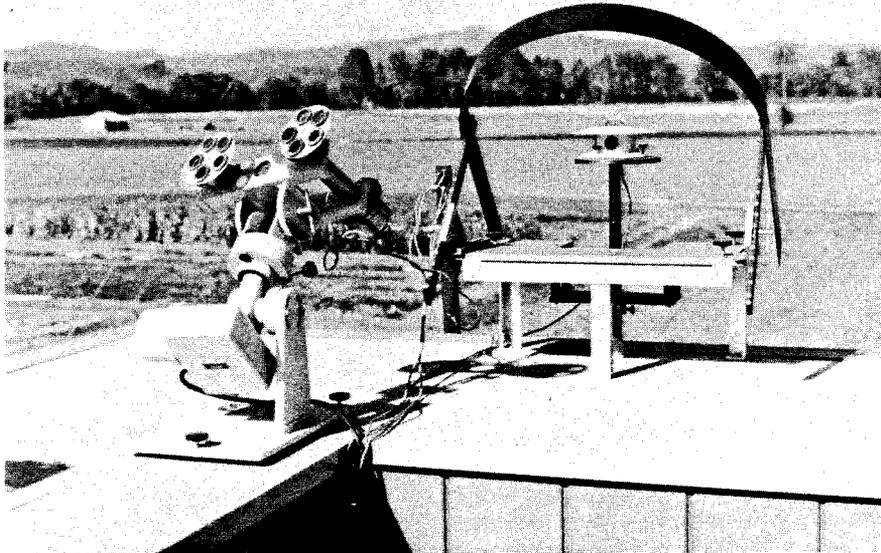


Fig. 4-21. Normal incidence pyrhemometers mounted on solar tracker (left) and the shadowband precision spectral pyranometer (right).

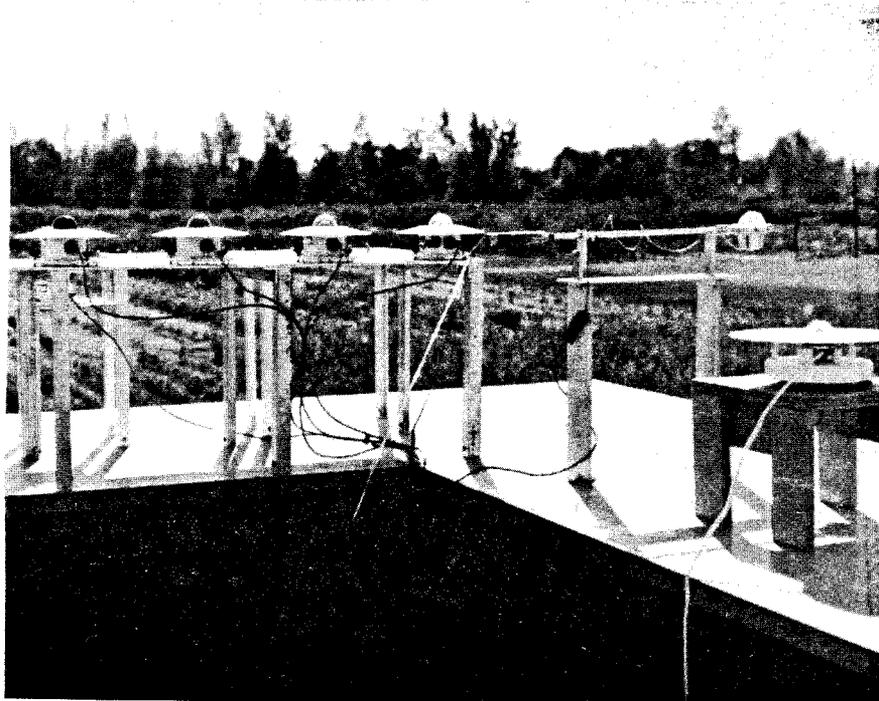


Fig. 4-22. Precision spectral pyranometers and the CSIRO Funk radiometer (extreme right, top row).

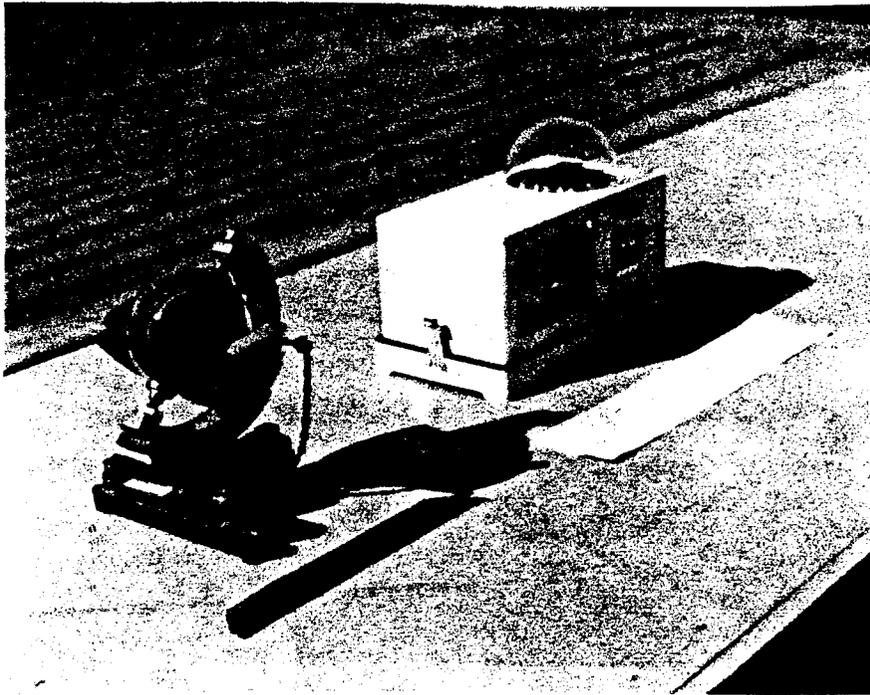


Fig. 4-23. The Campbell-Stokes sunshine recorder (left) and the Robitzsch bimetallic pyranometer (right).

Overview of Radiative Transfer Theory

Radiative transfer theory begins with an understanding of intensity and flux of radiation. Qualitatively, intensity can be thought of as a pencil or beam of radiation passing through an elemental area of dA and confined to a solid angle $d\omega$ associated with dA . If one integrates over all $d\omega$ on one side or the other of dA , one gets flux or flux density of radiation. A more complete explanation of intensity and flux of radiation can be found in Coulson (1975, pp. 1-4).

As a beam of radiation passes through the atmosphere, it can be absorbed and scattered (which includes reflection). In addition, radiation from atmospheric gases is emitted in the path of the beam. What radiation transfer theory does is to integrate the spectral intensity of radiation along a given direction taking into account absorption, scattering, and emission. Then integration is performed over all directions and over all wavelengths to find, for example, the flux of radiation at the surface of the earth from the atmosphere above. This is the radiant flux that would be observed using a pyrgeometer (after removing the upward flux). A brief development of radiative transfer theory is given by Coulson (1975, pp 4-6, Chap. 10.) More comprehensive treatments are given by Goody (1964), Liou (1980), and Elsasser (1960).

Solar and Terrestrial Absorption Spectra

The four principal absorbing constituents in the atmosphere are H₂O, CO₂, O₂, and O₃. Fig. 4.6 in Miller et al. (1983, p. 55) shows absorption for various gases. We can think of solar radiation entering the top of the atmosphere and being absorbed on its way to the Earth's surface. Thus the short wavelength end of solar radiation is completely absorbed by O₂ and O₃ in the upper atmosphere, long before it strikes the Earth's surface. The long wavelengths of solar radiation are partially absorbed in the troposphere.

We can think of terrestrial radiation as leaving the surface and passing upward through the atmosphere. Radiation beyond around 18 μ m is almost completely absorbed by the atmosphere. At shorter wavelengths, the amount of absorption is very dependent on wavelength. The atmosphere acts as an insulating blanket for terrestrial radiation, but there are some holes through which surface radiation can pass to outer space.

Rayleigh and Mie Scattering

The simplest atmospheric radiation model is that of a nonabsorbing medium in which the scattering particles are all small compared to the wavelength of radiation. This is the case when solar radiation passes through the atmosphere and is scattered by air molecules. This is called Rayleigh scattering (after Lord Rayleigh, who worked out the theory in the 1870's). The important aspect of Rayleigh scattering is that the amount of scattering is inversely proportional to the fourth power of the wavelength of incident radiation. Thus, blue color is scattered more than red color, which results in the blue sky.

When the scattering particles are of the same size or larger than the wavelength of incident radiation, then Mie scattering holds. Aerosols in the fractional micrometer to micrometer range thus result in scattering of all visible wavelengths of radiation, producing a white or milky appearing sky. The role of absorption of solar radiation by aerosols and hydrometeors is complex and not easily specified.

With regard to long-wave or terrestrial radiation, the chief interaction is absorption--scattering plays a minor role. Certain molecules like those of water vapor and carbon dioxide are strong absorbers at certain wavelengths as seen earlier. The absorption of long-wave radiation by aerosols is complex, while for hydrometeors it is essentially unity. In short, the interaction of short-wave radiation with atmospheric constituents is dominated by scattering, while for long-wave radiation, it is absorption. Geometric optics becomes important when solar radiation interacts with droplets.

Radiation Laws

There are four radiation laws that are relevant to atmospheric radiation measurements. They are Planck's Law, Kirchoff's Law, Stefan-Boltzmann Law, and Wien's Displacement Law. An adequate discussion of these laws is provided by Coulson (1975) so that here we only provide a few comments on each.

Planck's Law. This is a basic law from which other laws can be derived. It tells the magnitude of radiation that a black body or ideal radiation yields. It depends only on the temperature T of the radiating substance and the wavelength λ (or frequency ν) of interest.

The two forms of Planck's Law are

$$B_{\nu}(T) = \frac{2h\nu^3}{c^2[\exp(h\nu/kT)-1]}$$

and

$$B_{\lambda}(T) = \frac{C_1\lambda^{-5}}{\exp(C_2/\lambda T)-1} \quad .$$

The values of the constants are

$$h = \text{Planck's constant} = 6.6256 \times 10^{-27} \text{ erg s}$$

$$k = \text{Boltzmann's constant} = 1.3805 \times 10^{-16} \text{ erg K}^{-1}$$

$$c = \text{speed of light} = 2.998 \times 10^{10} \text{ cm s}^{-1}$$

$$C_1 = \text{first radiation constant} = \pi 2hc^2 \\ = 3.74150 \times 10^{-5} \text{ erg cm}^2 \text{ s}^{-1}$$

$$C_2 = \text{second radiation constant} = hc/k = 1.43879 \text{ cm K.}$$

$B_{\nu}(T)$ is the energy per unit time per unit area per unit frequency emitted into the hemisphere (2π steradians) above a surface at temperature T . $B_{\lambda}(T)$ is the energy per unit time per unit area per unit wavelength emitted with these same conditions. In order to get spectral intensity both $B_{\nu}(T)$ and $B_{\lambda}(T)$ need to be divided by π , the conversion between flux and intensity for isotropic radiators.

Kirchoff's Law. Whereas Planck's Law deals with black-body radiation (apparently nonexistent in nature), Kirchoff's Law deals with actual substances. It says that at a given wavelength, the ability of a substance to emit energy, as measured by its coefficient of emission ϵ_{λ} , is the same as its ability to absorb energy, as measured by its coefficient of absorption A_{λ} . Although our atmosphere does not

strictly conform to the conditions required for this law to be valid, they are sufficiently well met to make the law applicable everywhere except in the high upper atmosphere.

The formula is

$$A_{\lambda} = \epsilon_{\lambda} ,$$

which for a black-body surface becomes $A_{\lambda} = \epsilon_{\lambda} = 1$ for all wavelengths. A "gray-body" surface follows the relation

$$A_{\lambda} = \epsilon_{\lambda} < 1.$$

Wien's Displacement Law. This law can be obtained from differentiating Planck's Law with respect to wavelength and setting the result equal to zero. One then obtains an expression relating the wavelength (or frequency) of maximum intensity to the black-body temperature of a substance.

The equation for this law is

$$\lambda_{\max} T = \text{constant}$$

in which the value of the constant is 0.2897 cm K, if λ is in centimeters.

Stefan-Boltzmann Law. This law can be obtained by integrating Planck's Law over all wavelengths (in frequency) for a given temperature. The result is an expression that provides the energy per unit time per unit area emitted by a black-body substance. If one can assume a gray-body radiator (emission coefficient uniform with wavelength), then one has a very useful law relating flux of radiation to temperature.

For black-body radiation, the Stefan-Boltzmann Law is

$$E = \sigma T^{-4} ,$$

where $\sigma = 5.6697 \times 10^{-12} \text{ W cm}^{-2} \text{ K}^{-4}$. For gray-body radiation, the formula becomes

$$E = \epsilon \sigma T^{-4} ,$$

where $\epsilon < 1$ is wavelength-independent emissivity of a gray-body surface.

4.5.2 RADIATION TRANSDUCERS AND SENSORS

Types of Transducers

Thermopile. The thermopile is the most commonly used transducer for radiation sensors primarily due to the non-wavelength selective nature of its response. A thermopile is a series of thermoelectric junctions. These are junctions between two dissimilar metals. When alternate junctions are maintained at different temperatures, an electromotive force, known as the "Seebeck effect," is developed across them. The direction of current flow and the magnitude of the electromotive force produced is dependent on the material of the wires (e.g., copper-constantan, iron-constantan, and chromel-alumel) as well as the temperature difference between the junctions. When a thermopile is used as a radiation sensor, it can be shown that the temperature difference is proportional to the incident radiant energy; thus, the voltage developed by the thermopile is proportional to the incident radiant energy.

The Eppley wire-wound thermopile is a typical example of a thermopile transducer. It consists of a helix of constantan half of which has been copper plated, forming a series of copper-constantan junctions. One set of junctions is in thermal contact with a spectrally nonselective black absorbing surface, while the other set of junctions forms the reference at the temperature of a heat sink at approximately ambient air temperature and is imbedded in the body of the instrument. The electrical resistance of copper is about 3% of constantan; thus, the copper plated half of the helix acts as if it were made entirely of copper. The body of the instrument is shaded and has a high thermal inertia, ensuring that the cold junctions remain close to ambient air temperature.

Photoelectric transducers. Photoelectric transducers are highly wavelength dependent in their response but have a high sensitivity even at low levels of incoming radiant energy. They can be divided into two classes: those based on the internal photoelectric effect and those based on the external photoelectric effect.

Photovoltaic cells and photoconductive transducers are commonly used in meteorological sensors and employ the internal photoelectric effect. In photovoltaic cells a potential barrier exists in the semiconductor material (e.g., selenium and silicon). The barrier is created by the interface between adjoining areas of different doping (i.e., two different impurities have been introduced into the semiconductor material in concentrations of less than 1%). When the material is illuminated, the potential barrier separates the electric charges, one side becoming positive with respect to the other. This generates a measurable current and voltage proportional to the incident photon flux. In photoconductive transducers, the electrical resistance of the semiconductor material changes with radiative input. Thus, an external voltage must be applied with the change in current being proportional to the incident radiant energy.

Photocells and photomultipliers are radiation sensors based on the external photoelectric effect. Photomultipliers consist of a cathode-anode arrangement where incoming photons strike the cathode setting off a shower of electrons which are directed towards the anode because of the potential difference existing between the two. The number of electrons emitted by the cathode is amplified by a system of dynodes. Photomultipliers are used in high resolution, narrow spectral bandpass radiometers.

An example of a photovoltaic cell transducer is the Eppley TUVR. This radiometer uses a selenium barrier-layer photoelectric cell. A translucent quartz disc is used to reduce the incoming radiation to an acceptable level to insure long-term performance stability; the quartz window also provides close adherence to the Lambert cosine law. (The radiation passing through the quartz disc diminishes, according to the cosine of the zenith angle.) A narrowband (interference) filter further reduces the incident radiation amount and limits the spectral response to ultraviolet wavelengths (.295 to .385 microns). The unit is then calibrated under natural conditions of exposure by comparison with a standard ultraviolet pyranometer incorporating a thermopile sensor.

Mechanical transducers. The most common type of mechanical transducer utilizes differential expansion of bimetallic strips. Bimetallic strips are strips of metal with different thermal expansion coefficients that have been fused together.

The Robitzsch bimetallic pyranometer (actinograph) utilizes a bimetallic transducer. It consists of a pair of bimetallic strips, one blackened with a non-wavelength selective black paint and the other shaded and highly polished. Both bimetallic strips bend equally in response to ambient air temperature changes; however, the blackened strip absorbs incident radiant energy and bends to a greater degree. The differential bending between the two strips is translated through a mechanical linkage to a pen and chart recorder.

This type of transducer has numerous deficiencies, resulting in daily irradiation errors of +10% or more and although fairly simple to operate and maintain must be considered inadequate for many applications.

Accuracy of Radiation Sensors

The accuracy of radiation sensors (radiometer) is influenced by sensitivity (minimum radiation energy difference that produces a measurable output change), stability (the long-term change in calibration factor), temperature effects (variation of response as a function of ambient temperature), spectral selectivity (constant response at all wavelengths), linearity (constant proportionality between incident radiation and sensor response), aperture (for pyrhemometers, the amount of circumsolar radiation allowed by the

instrument's field of view), time constant (length of time required for a 63.2% response to step change in incident radiation), cosine response (the deviation of the directional response from that assumed), and the azimuth response (the deviation of the response due to the azimuth angle of orientation of the instrument). The WMO classification scheme is shown in the Table 4-6 below.

Table 4-6. The classification of accuracy of radiometers.

	(a) Sensitivity (mW cm ⁻²)	(b) Stability	(c) Temperature	(d) Selectivity	(e) Linearity	(f) Aperture	(g) Time constant (max.)	(h) Cosine response	(i) Azimuth response	Errors in auxiliary equipment		
		%	%	%	%			%	%	Gal- vano- meter	Milli- meter	Chrono- meter
Reference standard pyrheliometer	± 0.2	± 0.2	± 0.2	± 1	± 0.5	(1)	25 s	—	—	0.1 unit	0.1	0.1 s
<i>Secondary instruments</i>												
1st class pyrheliometer	± 0.4	± 1	± 1	± 1	± 1	(1)	25 s	—	—	0.1 unit	0.2	0.3 s
2nd class pyrheliometer	± 0.5	± 2	± 2	± 2	± 2	(1)	1 min	—	—	0.1 unit	± 1	—
										<i>Errors in recording apparatus</i>		
1st class pyranometer	± 0.1	± 1	± 1	± 1	± 1	—	25 s	± 3	± 3			0.3
2nd class pyranometer (2)	± 0.5	± 2	± 2	± 2	± 2	—	1 min	± 5-7	± 5-7			± 1
3rd class pyranometer	± 1.0	± 5	± 5	± 5	± 3	—	4 min	± 10	± 10			± 3
										<i>Errors due to wind</i>		
										%	%	
1st class net pyrradiometer	± 0.1	± 1	± 1	± 3	± 1	—	½ min	± 5	± 5			± 0.3 ± 3
2nd class net pyrradiometer	± 0.3	± 2	± 2	± 5	± 2	—	1 min	± 10	± 10			± 0.5 ± 5
3rd class net pyrradiometer	± 0.5	± 5	± 5	± 10	± 3	—	2 min	± 10	± 10			± 1 ± 10

The WMO international reference standard is the PACRAD III (Primary Active Cavity Radiometer) pyrheliometer. Other reference standard pyrheliometers are the Angstrom compensation pyrheliometer and the silver disk (Smithsonian) pyrheliometer. Several types of first class pyrheliometers are currently manufactured, including the Eppley normal incidence pyrheliometer (NIP).

No reference standard pyranometers have been developed, but several first class pyranometers are available, including the Eppley precision spectral pyranometer (PSP).

Standardization and Calibration of Radiometers

Prior to 1975, several different radiation scales were employed in the reference calibrations of radiometers. Since 1975 all measurements

have been made in accordance with the Absolute Radiation Scale or equivalently the World Radiometric Reference established at the International Pyrheliometric Comparison IV at Davos, Switzerland. Efforts have been made to rehabilitate data taken prior to 1975 to the new international scale; however, caution should be exercised when using data taken prior to 1975 to determine whether or not it has been rehabilitated.

Periodic calibrations are highly desirable in order to reduce potential long-term stability induced errors. Pyrheliometric calibrations should be made against reference standards traceable to PACRAD III. In the laboratory, pyranometers are calibrated using an integrating sphere equipped with an incandescent light source.

The performance of a pyranometer can also be checked in the field by the shading method. The method consists of making simultaneous measurements under clear skies with a pyrheliometer and the pyranometer, with the latter being alternately exposed to and shielded from the direct solar radiation; an occulting disc subtending the same solid angle at the pyranometer as the field of view of the pyrheliometer is used for this purpose. The difference between the unshaded and shaded pyranometer readings gives, for all practical purposes, the vertical component of the direct normal solar radiation. This, in turn, is checked against independent measurements from the pyrheliometer, with the vertical component of the direct normal solar radiation given by the product of the pyrheliometer reading and the cosine of the solar zenith angle.

4.5.3 APPLICATIONS

Associated Instrumentation

Broadband spectral measurements. The spectral response of a pyranometer or a pyrheliometer is generally governed by the transmission characteristics of the material (glass/quartz) of the protective covering used to minimize extraneous effects (e.g., convective heat loss). With the materials generally used, the instruments detect radiation over the spectral interval from ≈ 0.3 to $3.0 \mu\text{m}$; however, when broadband spectral measurements are desired, absorption-type glass filters are used. WMO recommends the use of Schott glass filters with designated lower cut-off wavelengths. Some of the commonly used filters with their designations and passbands are listed below:

WG 295:	from ≈ 0.290 to $2.9 \mu\text{m}$
OG 530:	from ≈ 0.530 to $2.9 \mu\text{m}$
RG 630:	from ≈ 0.630 to $2.9 \mu\text{m}$
RG 695:	from ≈ 0.695 to $2.9 \mu\text{m}$.

By taking the differences of measurements made with pyranometers (pyrheliometers) fitted with appropriately chosen filters, irradiations

over narrow spectral intervals can be determined. The lower cut-off wavelengths of these filters exhibit temperature dependence and may vary by $\pm 0.01 \mu\text{m}$ over the range of temperatures typically encountered. Narrowband interference filters are generally used in radiometers with the highly sensitive photoelectric detectors.

The total (short- and long-wave, ≈ 0.3 to $50 \mu\text{m}$) radiation from the sun is measured with a pyrrometer. In the Funk instrument, the upward-looking portion of the thermopile is covered with a protective polythene dome, which transmits from ≈ 0.3 to $40 \mu\text{m}$; the downward-looking portion of the sensor is enclosed in an aluminum cup blackened on the inside; the temperature of the cavity is continuously monitored. The voltage issuing from the instrument is proportional to the difference between the short- and long-wave radiation incident upon the upward-looking part of the thermopile sensor and the black-body emission by the cup, which can easily be calculated. The incoming short- and long-wave radiation is then easily determined. The instrument generally has different calibration factors in the short- and long-wave regions. It is easily converted to a net pyrrometer by replacing the blackened aluminum cup with a polythene dome.

Solar tracking, occulting discs and the shadow band. Pyrheliometers must always be aimed in the direction of the sun in order to measure the direct normal solar radiation; thus solar tracking is necessary. A solar tracker is a clock-driven device that has manual adjustments for solar declination (elevation adjustment) and local meridian passage time (azimuth adjustment). When the solar tracker is initially mounted, it must be levelled and oriented to true north and set to the latitude of the station. A pin-hole imaging device is used to align the pyrheliometer with the sun initially. The relatively large field of view allows for slight tracking inaccuracies.

Occulting discs are available to obscure the sun from a pyranometer covering the same field of view that is seen by a pyrheliometer yielding a measure of the diffuse radiation component. The initial set-up requirements and adjustments for the occulting disc are similar to that of the solar tracker.

Another method of obtaining diffuse radiation measurements is to employ a shadow band. The shadow band consists of a rigid metal band that obscures the sun throughout the day. Declination adjustments are necessary, and the initial setup is similar to that of the solar tracker. A correction factor for the excess diffuse (sky) radiation obscured must be applied to the measurement, which depends on the season and latitude and ranges from 4 to 25 percent.

It is possible to compute the diffuse radiation (D) from pyranometer and pyrheliometer measurements via

$$D = G - S \cos \theta_0$$

where G is the global radiation measured by the pyranometer, S is the direct normal radiation from the pyrhelimeter, and θ_0 is the average solar zenith angle during the measurement.

Sunshine duration recorders. The duration of sunshine is a measure of the amount of time that the direct solar irradiance is above a certain threshold value over the period from sunrise to sunset. Several design variations have been used, including thermometric and photoelectric switching, tracing a beam on photographic paper, and focusing the sun's rays with a glass sphere onto a paper chart. It is the latter technique that has become the WMO reference standard sunshine duration recorder and is known as a Campbell-Stokes sunshine recorder. The resolution of measurements is to the nearest tenth of an hour in accordance with accumulating practices set forth by WMO. The average threshold irradiance for the Campbell-Stokes sunshine recorder is 210 W m^{-2} but depends on a number of factors, including atmospheric turbidity and moisture content of the card. A convenient feature of the Campbell-Stokes sunshine recorder is that after initial levelling and orientation, no further actions are necessary except to change the cards daily and to keep the glass sphere clean.

Exposure and Siting Requirements

Ideally, pyranometers should be mounted such that no obstructions of any kind extend above the plane of the sensing element, and the instrument is accessible for frequent cleaning. In practice, the horizon should not exceed 5° , especially from east-northeast, through south, to west-northwest. No shadows should be cast at any time on the instrument, and no reflecting structures such as walls should be nearby. A 5° horizon will obstruct only about 1% of the global radiation and thus can be ignored. For larger obstacles, corrections can be applied under the assumption of isotropic sky radiation.

Sampling

The integration time available to a radiation sensor to make a measurement must be sufficiently long compared to its time constant; this fact should be borne in mind while using a filter-wheel arrangement in conjunction with a pyrhelimeter to make multi-wavelength measurements.

General Applications

In this section we consider some of the more obvious applications of radiation information.

Satellite meteorology. Meteorological satellites have been used since April 1, 1960, when TIROS I was launched. The primary sensors were TV cameras and infrared radiometers. Since that time many other meteorological satellites have been launched, including both polar orbiting

and geostationary satellites. Remote sensing from these satellites is used to determine earth surface temperature, temperature and humidity profiles, wind and earth imagery.

The idea behind these measurements is to look at the earth and its atmosphere through various portions of the electromagnetic spectrum. For example, radiation sensed in the 15 μm CO_2 band can be used to construct a temperature profile. This requires measurements of radiation from six to eight quite distinct frequencies within this band combined with a mathematical inversion technique (Deepak, 1977). In the same way, one can choose a wavelength that is poorly absorbed by the clear atmosphere so that radiation from the earth's surface is transmitted up to satellite level. Using Planck's Law, the earth's surface temperature can be estimated.

Energy sources. Since the energy crunch in 1973, there has been a steady increase in the use of solar energy. Numerous industries have built rooftop flat-plate solar collectors as a source of thermal and electrical energy. The same phenomenon is also occurring in house construction.

Architecture. The design of houses has been influenced by taking into account the sun's rays. The general theme is to maximize the collection of solar energy during winter months and minimize its absorption during hot summer months. The methods include preferential location and size of windows, the use of berms and overhangs, and shading by trees. This is the passive use of solar energy. The most active uses of solar radiation are to heat water for domestic consumption and to heat house air using a heat exchanger.

Agriculture. Of course, the most important use of solar radiation in agriculture is in the photosynthesis process, without which there would be no food or fiber production. Also, in springtime, sufficient solar radiation must be absorbed by the soil to initiate germination after planting.

Surface heat budget. In the absence of frontal passages, the dominant factor controlling the daily cycle of temperature in the biosphere is the short-wave radiation received during the day and the long-wave radiation emitted to the upper atmosphere and space during nighttime. The presence of overcast conditions during daytime or nighttime greatly dampens the amplitude of the daily cycle. An incorrect forecast of cloud amount can have a severe impact on high or low temperature forecasts.

Air pollution. One of the more deadly results of solar radiation is the formation of toxic gases through photochemical association and dissociation. Of these the most common is the formation of ozone, resulting from the dissociation of oxygen molecules into atomic oxygen and their recombination with diatomic oxygen with the aid of photons.

4.5.4 QUESTIONS AND LABORATORY EXERCISES

Questions

1. The rotational bands of water vapor lie primarily in the _____ portion of the electromagnetic spectrum.
 - a. infrared
 - b. near-infrared
 - c. visible
 - d. ultraviolet
 - e. cosmic ray

2. The energy of a photon is given by Planck's constant times.
 - a. the wavelength
 - b. the reciprocal of the wavelength
 - c. the speed of light
 - d. the speed of light divided by the wavelength
 - e. the frequency divided by the speed of light

3. Differentiating and integrating Planck's law with respect to _____ yields the Wein displacement and Stefan-Boltzmann laws respectively.
 - a. T, temperature
 - b. λ , wavelength
 - c. h, Planck's constant
 - d. ϵ , emissivity
 - e. none of the above are correct

4. The earth's average surface temperature is approximately 15°C. If it were to increase to 30°C, the radiation emitted by the surface would increase by a factor of _____. (Assume black-body conditions.)
 - a. 1.00, i.e., remain unchanged
 - b. 1.23
 - c. 2.00
 - d. 4.00
 - e. 16.00

5. Some snakes are sensitive to infrared radiation. The normal temperature of humans is 37°C. What wavelength of infrared radiation should a people-eating snake's sensors be "tuned" to?
 - a. 78.3 μm
 - b. 0.11 μm
 - c. 9.3 μm
 - d. 0.01 μm

6. Which of the following is the largest source of energy in the atmospheric energy budget?
- the absorption of solar radiation
 - the release of latent heat
 - the emission of terrestrial radiation
 - the absorption of terrestrial radiation emitted by the earth's surface
7. Which of the following is in approximate radiative equilibrium when averaged over the whole earth and averaged over time (decades)?
- the planet earth (earth-atmosphere system)
 - the atmosphere
 - the earth's surface
 - all are in approximate radiative equilibrium
8. A planet with a black-body radiative temperature of 400°K and an albedo of 50% is located 5.00×10^{15} m from the nearest star. The planet's solar constant is
- 1450 W m^{-2}
 - 2900 W m^{-2}
 - $8.21 \times 10^{11} \text{ W m}^{-2}$
 - $11,600 \text{ W m}^{-2}$
 - none of the above
9. Which of the following is not a property of a thick cloud layer in the atmosphere?
- clouds increase the local albedo
 - clouds reflect the terrestrial radiation emitted by the surface back towards the surface
 - clouds absorb a portion of the incident solar radiation
 - cloud tops emit terrestrial radiation of which a portion reaches space
10. The condensation of one cubic meter of water vapor to form precipitation or cloud droplets releases enough latent heat to melt _____ cubic meter(s) of ice.
- 6.7
 - 1.9
 - 1.0
 - 0.51
 - 0.15
 - none of the above

11. The atmospheric "window" between 3.0 and 4.5 μm is not as significant as the 0.3 to 1.0 and 10 to 12 μm windows because
- only methane absorbs strongly in that region
 - the sun and the earth emit very little radiation in that band
 - the black-body assumption for thick clouds breaks down at wavelengths less than 8 μm
 - all of the above are correct.
12. Kirchoff's Law is best expressed as
- λ^{-4}
 - σT^4
 - $a_\lambda = \epsilon_\lambda$
 - $\epsilon = a = 1$
13. During periods of clear weather and light wind conditions, an airplane will most likely encounter turbulence during takeoff and landing at about
- 6:00 a.m.
 - 3:00 p.m.
 - 9:00 p.m.
 - 12:00 a.m.
14. List the values for the following parameters if the object obeys the black-body hypothesis:

$$\epsilon_\lambda =$$

$$a_\lambda =$$

$$r_\lambda =$$

$$t_\lambda =$$

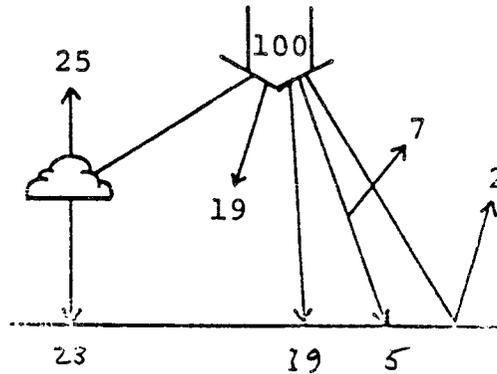
$$E^* = \quad , \text{ at } T = 1.00^\circ\text{K}$$

Define the gray-body hypothesis.

15. Suppose that a cosmic dust cloud drifted between the earth and the sun reducing the measured solar constant to 1104 W m^{-2} . Resulting colder temperatures reduce evaporation and cloudiness, but increase the amount of ice at the earth's surface. The planetary albedo changes to 38% before an equilibrium is reached. Calculate a new equivalent black-body radiative temperature for the earth during this ice age climate.
16. In some future time the solar constant for Jupiter will be 13.5 W m^{-2} . The average distance of Jupiter from the sun is $7.77 \times 10^{11} \text{ m}$. At what color will the sun's maximum emission occur? (You may assume that the sun remains the same size.)
17. Using radiation arguments, explain the probable meteorological conditions accompanying Norman's (OK) record lowest-high and highest-low temperatures in winter.
18. From our energy budget diagram, list five inputs (i.e., sources of energy) to the atmospheric budget.
19. In words and/or symbols list the energy budget equations for the planet, the atmosphere, the earth's surface.
20. Extra Credit (5 points) Strip charts on solar radiation measuring devices depict a range of 0 to about 2000 W m^{-2} . Give an example of how the surface solar irradiance can exceed the solar constant.
21. Briefly discuss the following:
 - a. Clouds act to moderate the diurnal temperature range, i.e., daytime highs are cooler, nighttime lows are warmer. Why?
 - b. How can a planet closer to the sun than the earth have a lower effective black-body radiating temperature?
22. Suppose that a star radiates as a black body with a temperature of 8000°K . Assume that the star has the same radius as our sun.
 - a. Calculate the star's wavelength of maximum emission.
 - b. Calculate the "solar constant" for a planet located $3.0 \times 10^{11} \text{ m}$ from the 8000°K star.

23. Past estimates of the earth's albedo have ranged from 34 to 50%. Calculate the earth's black-body radiating temperature if its albedo was indeed 50%.
24. Given that the solar constant measured outside the earth's atmosphere at a distance of 1.5×10^8 km from the sun is $2.00 \text{ cal cm}^{-2} \text{ min}^{-1}$,
- Calculate the luminosity (or total radiant energy output) of the sun.
 - The diameter of the sun is approximately 1.39×10^6 km, calculate the radiant energy emitted by a cm^2 of the sun's surface.
 - Given that the Stefan-Boltzmann constant is $8.132 \times 10^{-11} \text{ cal cm}^{-2} \text{ min}^{-1} \text{ K}^{-4}$, obtain the effective black-body radiating temperature of the sun.
 - What is the wavelength of maximum emittance that can be computed from the temperature obtained in part c? (Recall Wein's constant is 0.2897°K cm .)
 - If the earth had no atmosphere, what would be the average solar radiation reaching the surface during a 24-h period?
25. What factors are important in determining how much of the sun's energy actually reaches a cm^2 of the earth's surface?
26. Calculate the average irradiance of solar radiation reaching the earth's orbit at its mean distance from the sun. Assume the sun radiates as a black body with $T = 5780^\circ\text{K}$.
27. The distance from the earth to the sun varies from approximately 1.48×10^8 km in early January to 1.52×10^8 km in early July. Calculate the percentage difference in the "solar constant" between early January and early July.

28. (4 points) From the solar radiation diagram below,



atmospheric transmissivity = _____

atmospheric absorptivity = _____

atmospheric reflectivity = _____

planetary albedo = _____

Laboratory Exercises

Laboratory Exercise #1: Response Time of Radiation Sensors

Objective:

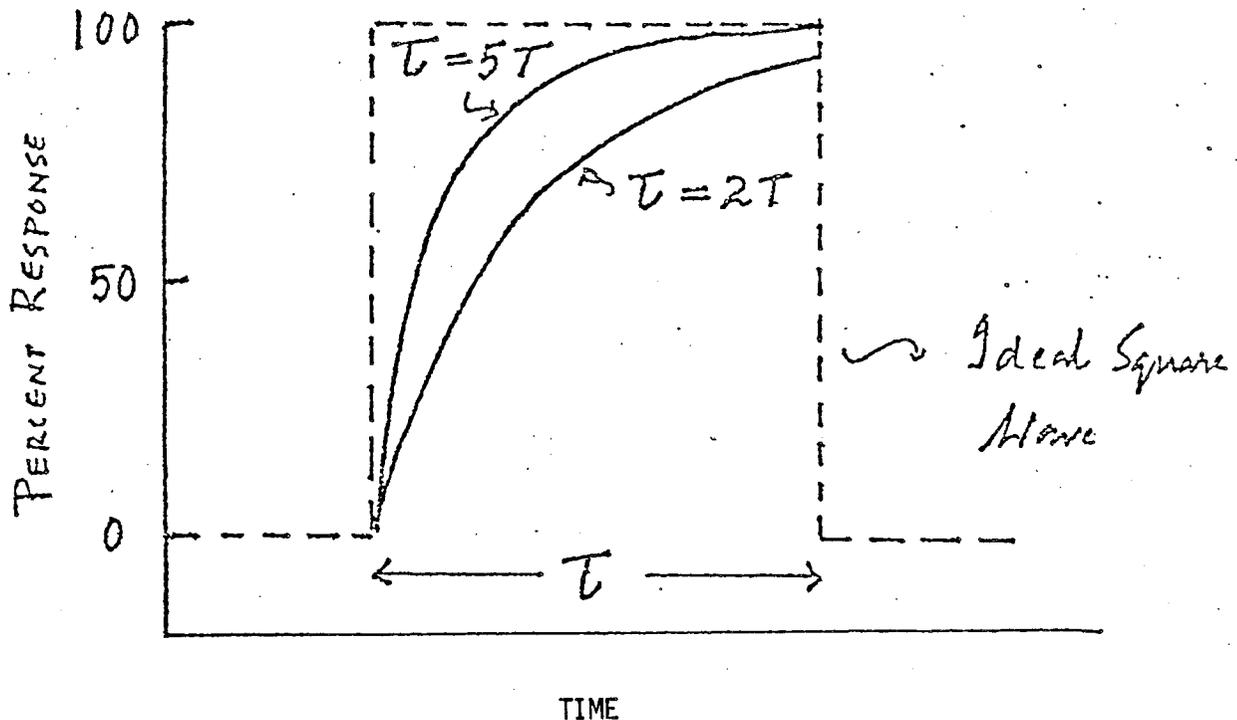
Determine the time constant for a pyranometer(s).

Equipment:

- Pyranometer(s). If more than one is to be tested, it is suggested that they be from different manufacturers.
- Square wave generator (such as TEK DC 503 counter/timer and TEK FG 501 signal gen.)
- Strip chart recorder (such as MFE Dual Trace Recorder)
- 24V dc power supply
- Relay (normally open)
- 150W flood lamp
- Transistor 2222A, resistor 4.7 K Ω , capacitor .1 mf

General:

The time available to a radiation sensor to make a measurement is known as the integration time. The integration time required to make a truly representative measurement can be expressed in terms of the time constant of the sensor. Assuming that the level of the signal being detected remains the same over the integration time, the concept of the time constant can be understood by examining the response of the sensor to an ideal square wave pulse of width τ .



It can be shown that the response of the sensor at time t after the square-wave signal has been switched on is $100(1 - e^{-t/T})\%$ of full scale response, with T being defined as the time constant of the sensor. It is easily seen that T is the time required by the sensor to indicate 63.2% of full scale response. The integration time should at least be five times the time constant in order that the sensor response may be 99.3% of full scale. Another characteristic of the sensor, called the rise time T_r , is defined as the time required for the sensor response to change from 10 to 90% of full scale, $T_r = 2.2T$.

Instrument:

The laboratory setup is shown in Fig. 4-24. The device under test (Eppley pyranometer; Solarimeter) can be subjected to either steady illumination or periodic illumination at different frequencies.

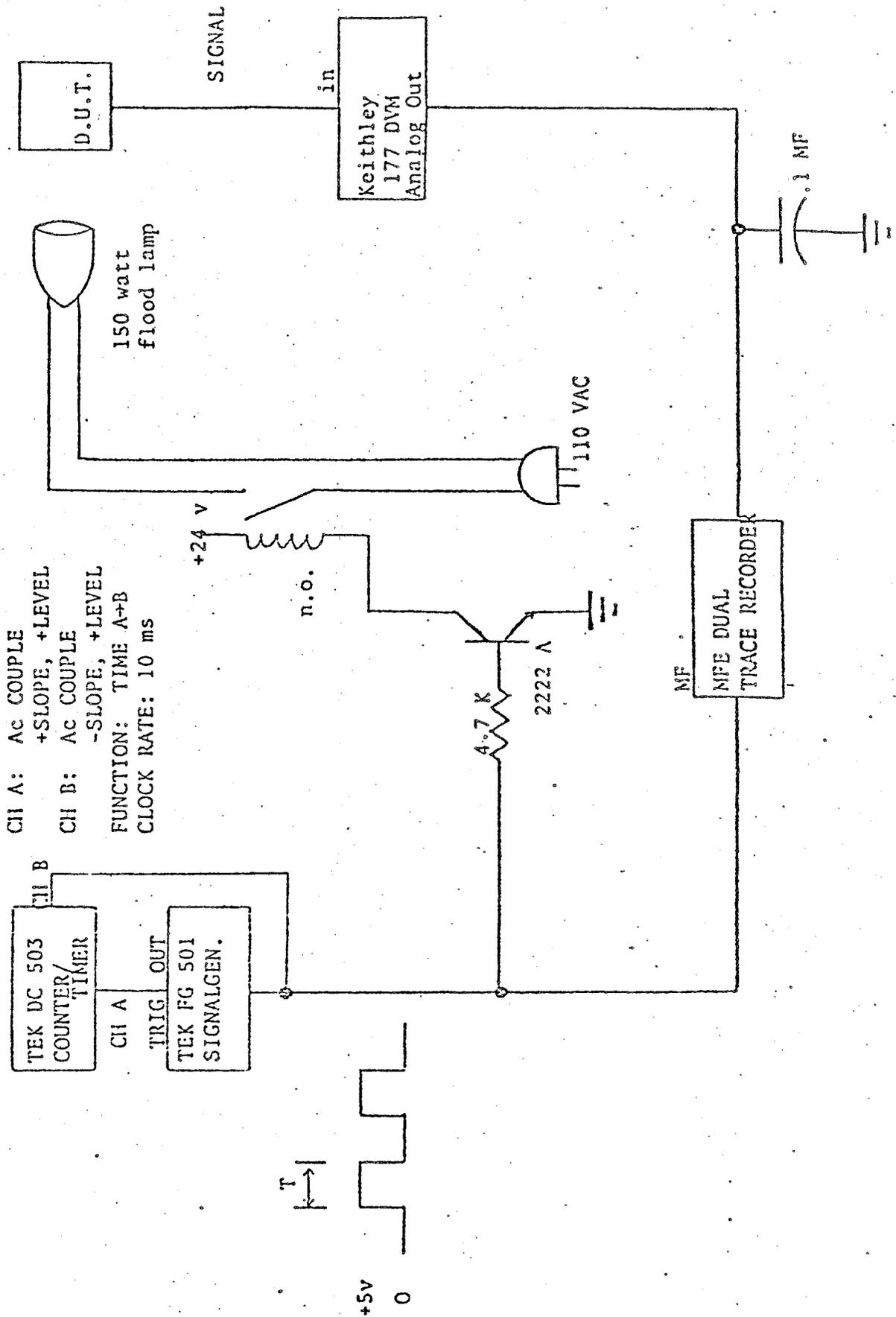
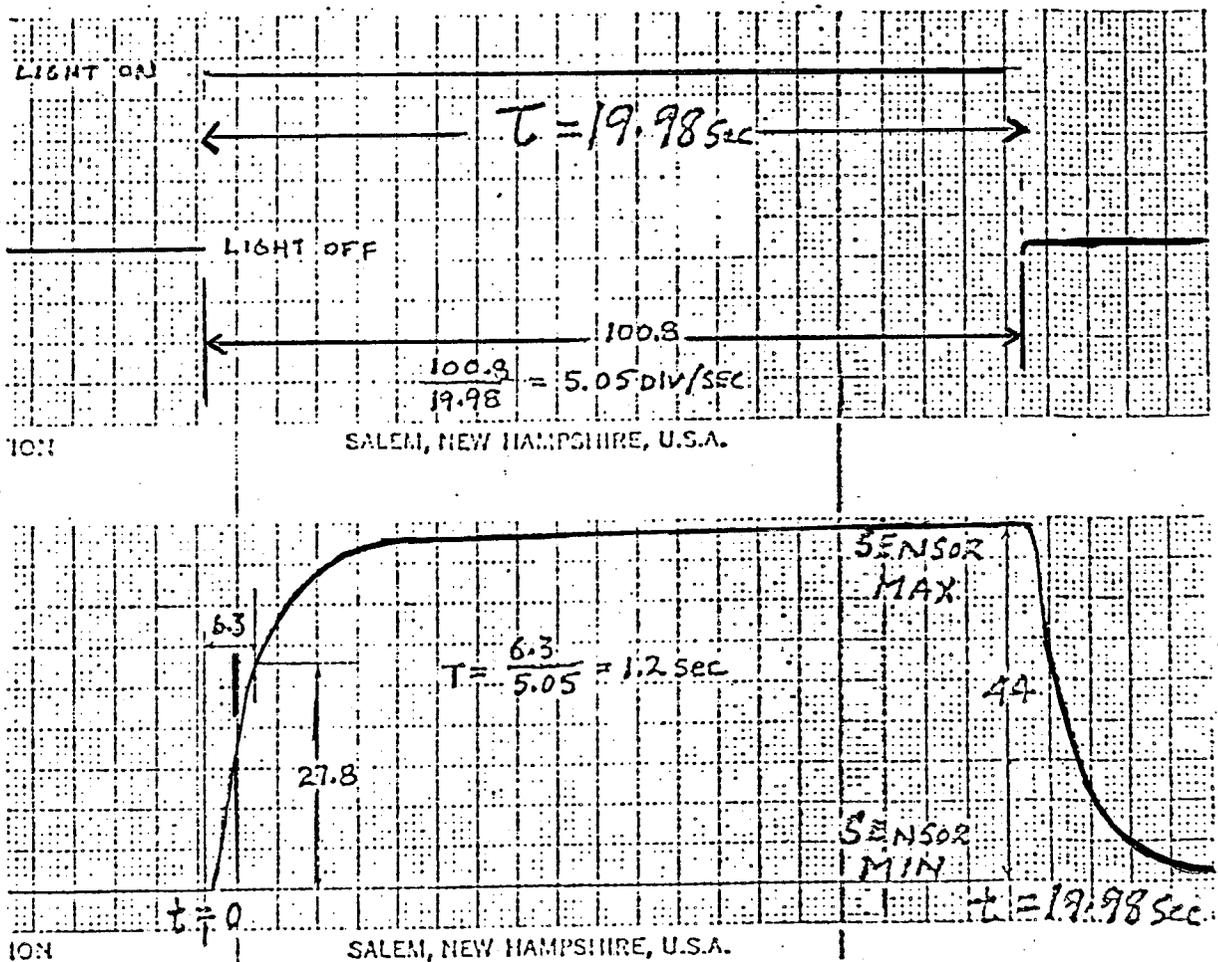


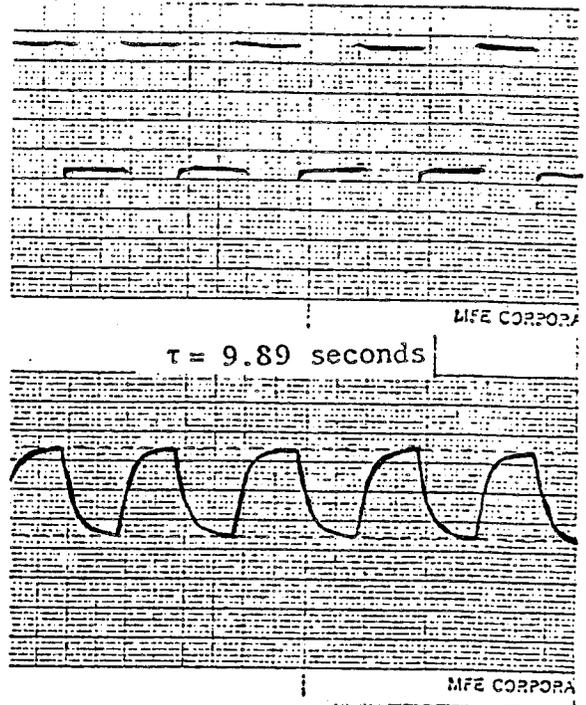
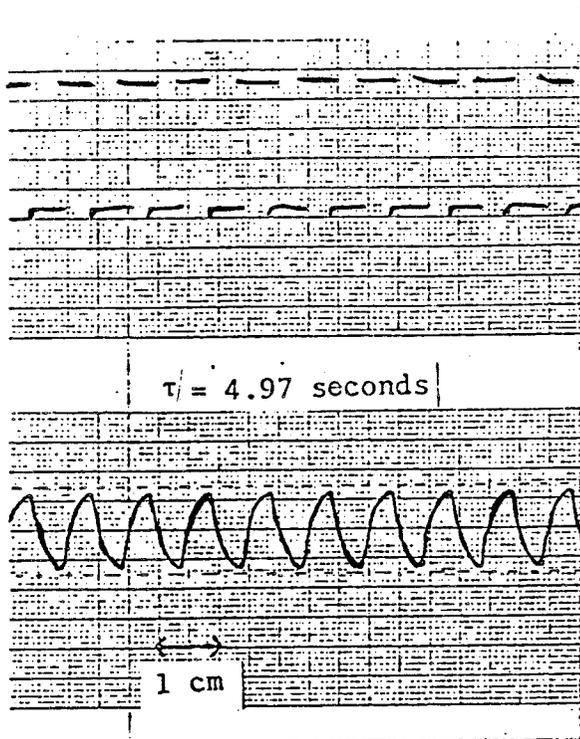
Fig.4-24. The experimental setup for the measurement of response times.

Method:

Shown below is a facsimile of a chart record obtained when an Eppley pyranometer was exposed to a light source controlled by a square-wave pulse of width $\tau = 19.98$ s.



The method of determination of the time constant, as indicated on the chart, is self-evident. Knowing that the full scale response is 44 units (arbitrary), the time constant is determined as the time that should elapse after the light is switched on for the sensor response to be 63.2% of 44; it can also be derived from the rise time, T_r . Elapsed times are determined in terms of chart length, assuming the chart speed remains uniform.



Nominal chart speed = 1 mm/sec

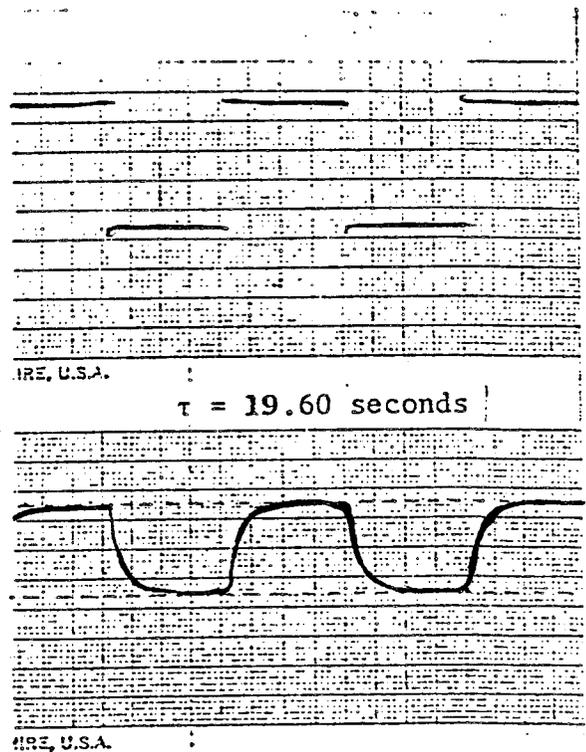
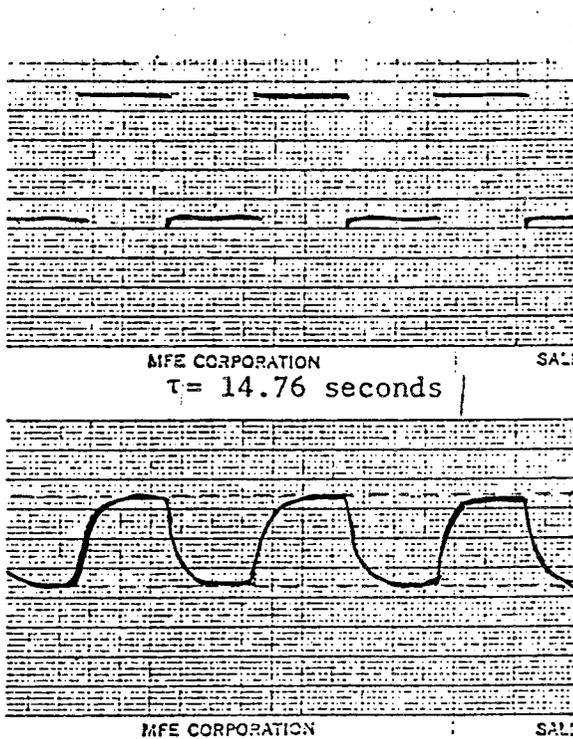


Fig. 4-25. Response of the Eppley normal incidence pyrhelimeter to periodic illumination.

Laboratory Exercise #2: Measurements of Solar Radiation

Objective:

Make measurements of global and diffuse radiation to determine direct solar radiation and the local albedo of the Earth's surface.

Equipment:

- Pyranometer
- Occulting disk
- Millivolt recorder
- Apparatus for inverting pyranometer.

Background:

The Eppley Model 8-48 measures the flux of short-wave radiation impinging on a horizontal surface. Flux is given in (energy time⁻¹ area⁻¹). The total or global solar radiation that is normally measured by a pyranometer consists of two parts: the direct solar beam, which is essentially parallel radiation that is transmitted directly through the atmosphere and diffuse radiation from the sky. The latter component contains radiation that has been scattered from the solar beam on its initial downward traverse of the atmosphere and that which has been reflected from the surface and returned to the downward direction by the overlying atmosphere.

The flux of direct solar radiation on a unit horizontal surface is

$$Q_n \cos \theta = (Q + q) - q$$

where θ is the zenith angle of the Sun, Q_n is the flux of direct radiation on a unit surface normal to the solar beam, $(Q + q)$ is the global flux, and q is the diffuse flux. In our experiment we will measure $(Q + q)$ and q to get $Q_n \cos \theta$, then determine $\cos \theta$ to get Q_n .

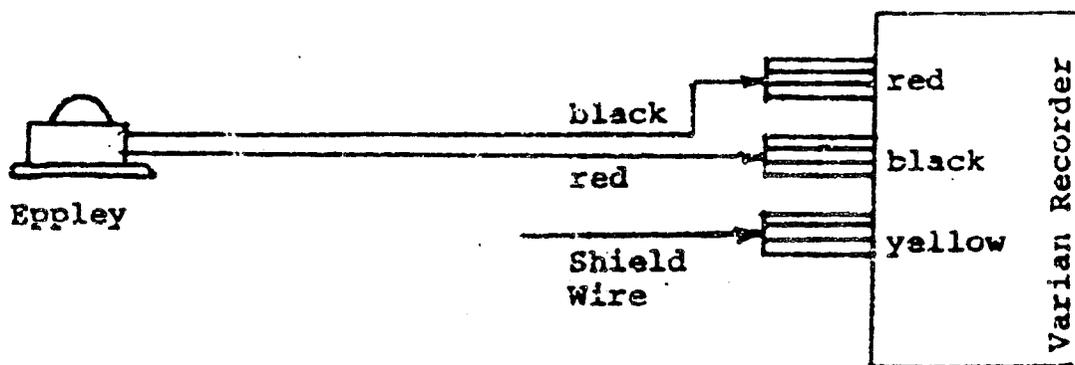
In addition, rotating the radiometer so that it points toward the nadir allows us to measure the scattered and reflected solar radiation by the Earth's surface, namely, $(Q + q) \alpha$, where α is the surface albedo. Since $(Q + q)$ has already been found α can be computed.

Measurement procedure:

Site selection and equipment setup:

- Select a grassy area where there is good exposure to the direct rays of the Sun. You need a perfectly clear sky in the direction of the sun to do this experiment.

- Level the cross-arm attached to the tripods using the torpedo-level and the height-adjusting knobs on the support posts. Use tissue paper (e.g., Kleenex) to clean the globe of the radiometer. Then carefully place the Eppley on the center mount and use the aluminum thumb screws to loosely attach it to the mount. Level the Eppley using the black thumb screws and, finally, tighten the aluminum screws so that the radiometer is firmly fixed to the mount.
- Attach one end of the instrument cable to the Eppley, the other end to the Varian. (The Varian is the manufacturer's name of a millivolt recorder. Any suitable millivolt recorder can be used for this experiment.) The red lead of the cable goes to the black terminal on the recorder, the black lead to the red terminal. Connect the bare shield wire to the yellow terminal. A diagram of the connections is given below.



- Set the Varian to standby and then turn on the power. Switch the pen lift to off and carefully remove the red plastic pen cap. (NOTE: Please, please do not lose the pen cap!) Switch the pen lift to on. Set the chart speed to 10 cm/min.

In order for the left zero line on the chart to correspond to zero voltage, press zero read and use the zero pos(ition) control to get the pen trace exactly on the zero line. Return the recorder to standby and set the chart speed to 2.5 cm/min. You are now ready to make radiation measurements.

Radiation measurements:

- Sketch the physical setup and show the distances to trees, buildings, etc.
- The first measurement is global radiation. Set the recorder to norm(al) and adjust the span switches to get the maximum deflection without the pen going off scale (start with 10

mV/FS). Note the full-scale voltage on the chart for this and all measurements. With personnel cleared from around the sensor (this goes for any measurement) record one or two minutes of data. Note the local time on your chart for this and all other measurements.

- The second measurement is diffuse sky radiation. Adjust the shield and arm on the third tripod so that the sensing area of the Eppley is shielded from the direct rays of the sun. Record one to two minutes of data.
- The third measurement is reflected and scattered surface radiation. Remove the upper knurled knobs (don't disturb the lower ones) and invert the cross-arm. The Eppley should be facing downward and still exactly horizontal. Record one to two minutes of data.
- The fourth measurement is, again, global radiation. Invert the cross-arm so that the Eppley faces upward. Make sure the instrument is level. Record one to two minutes of data.

Zero check:

- Change the chart speed to 10 cm/min, press zero read and record a few cm of the pen trace. The pen trace should coincide with the zero line. If it doesn't, then an adjustment will have to be made to the data you've collected. Return the recorder to standby. Set the pen lift to off and replace the plastic pen cap. This completes the experiment. Tear off the chart paper and submit it (or a xerox copy) with your report.

The instrument calibration for the pyranometer is $11.35 \mu\text{V/W m}^{-2}$.

- Compute the average global radiation ($Q + q$) in W m^{-2} (use these units for subsequent calculations) for the two measurement periods and the average for the experiment. The values for each period should be only slightly different.
- Compute the average diffuse sky radiation (q).
- Compute the average direct solar radiation on a horizontal surface ($Q_n \cos \theta$) from the values obtained above.
- Compute the average direct solar radiation on a surface normal to the beam (Q_n). The zenith angle θ that applies to the time of your experiment can be determined from the appropriate sun-path diagram in Table 170 of the Smithsonian Meteorological Tables after reading the introductory material.
- The flux of short-wave radiation measured with the Eppley facing downward is $\alpha(Q + q)$ where α is the local albedo. Compute the value of α .

- If all the power output from a 100W light bulb could be focused on a 1 m^2 area, how many light bulbs of this size would be required to match the solar radiation ($Q + q$) that you measured?

Laboratory Exercise #3: Field Calibration of a Pyranometer Using a Pyrhelimeter (Shading Method)

Objective:

Calibration of a pyranometer using a pyrhelimeter and an occulting disk.

Equipment:

- Pyranometer
- Pyrhelimeter with solar tracker or other suitable alignment device
- Occulting disk with same solid angle dimension as field of view
- Relatively unobstructed site with direct sunshine
- Voltmeter (preferably digital) or other recording device
- Known calibration constant for the pyrhelimeter
- Clock with correct local standard time

Procedure:

Set up the Eppley Normal Incidence Pyrhelimeter and the Eppley Precision Spectral Pyranometer for routine measurements under clear skies. Each sequence of measurements consists of the following:

- Record the pyranometer signal V_G and the pyrhelimeter reading V_C at time t .
- Shade the pyranometer from the sun using the occulting disk; record the pyranometer signal V_d at $(t + 10 \text{ s})$ while the disk is still in place; record the pyrhelimeter reading at the same time.
- Remove the occulting disk; record the pyranometer and pyrhelimeter readings at $(t + 20 \text{ s})$.

Under normal clear skies, especially when there are no clouds anywhere in the vicinity of the sun, the two values of V_G and the three values of V_C should be very close to one another. Using the average values of V_G and V_C , the pyranometer calibration constant is calculated from the expression

$$K_S = \frac{(V_G - V_d)}{V_C \cos \theta_0} K_C \quad ,$$

where

K_C = calibration constant for pyrhelimeter

θ_0 = solar zenith angle at time $(t + 10 \text{ s})$ or, for all practical purposes, at time t .

Repeat this procedure for at least five different values of t over a 1-h period.

4.6 PRECIPITATION

4.6.1 INTRODUCTION

Discontinuity of Precipitation

Precipitation is spatially and temporally discontinuous.

Data

Data required as intensities/rates and amounts.

Snow

Snow presents additional problems to those encountered in the measurement of liquid precipitation. It is much less dense than liquid precipitation, and thus is much more subject to effects of turbulence in the vicinity of the gage orifice. Also because it is solid, it may collect on or around the gage orifice and so change the aerodynamic characteristics and collection efficiency of the gage. Because it is often stored on the ground surface as a frozen reservoir of water and evaporative losses at the time are minimal, measurements can be made for some time after the event has ended. Three basic measurements are used in the quantification of precipitation in the form of snow.

1. Snowfall (S_f): This is the depth (cm) of fresh snow that falls on an even horizontal surface during an event, or the sum of such values for individual events over any given time period. It is often used to approximate the water equivalent of the snowfall, by multiplying by 0.1, using the assumption that the density of freshly fallen snow is 100 kg/m^3 .

2. Snowfall water equivalent (S_w): The equivalent depth (mm) of the snowfall when the catch is melted. This is necessary to combine snow data with liquid precipitation data. These values are obtained in a variety of ways, including: (a) from $\text{snowfall} \times 0.1$ (e.g., Canada AES), (b) from $\text{snowfall} \times k$, where k is a function of temperature during snowfall, and (c) melting and weighing snow catch in a gage or on a snowboard.

3. Snowcover (S_c): The depth (cm) or water equivalent (mm) of the total snow on the ground at the time of measurement. Obtained by coring the snowpack, and melting or weighing the catch, or by using a measuring rod to determine snow depth.

Acid-base Aqueous Chemistry

Students are expected to be familiar with the following concepts, which can be found in most freshman chemistry texts.

Equilibrium
Stoichiometry
Molar concentration, M
Standard electrode potential

4.6.2 THEORETICAL BACKGROUND FOR PRECIPITATION INSTRUMENTATION

Liquid Precipitation--Rain, Dew

1. Spectra (number/size).
2. Rainfall rates.

Solid Precipitation--Snow, Hail

In recording precipitation, distinction is made in general only between rain, snow, and total precipitation. In special cases other solid precipitation forms are distinguished. One such case is the use of hail pads to monitor the occurrence of hail in agricultural areas sensitive to its impacts (Changnon et al., 1977).

All forms of precipitation when measured in terms of equivalent depth, are given with units of mm (or cm) or inches (in hundredths). Conversion between these units is according to 1 inch = 25.4 mm. More information about the units used and preferred resolutions of readings can be obtained from Middleton and Spilhaus (1960) and WMO (1971).

1. Density of snowfall vs temperature relationship: The air space in newly fallen snow, and hence the density of the snow, is a function of the type of snowfall, the nature of the crystal structure, and the meteorological conditions during the event (particularly temperature and wind speed). Changes in ice crystal structure and their aggregation can be closely related to air temperature at the time of the precipitation event (e.g., Schemenauer, Berry, and Maxwell, 1981), and empirical relationships between temperature and snow water equivalent have been established for many locations. In Canada the generally used relationship is

$$S_w = k * S_f \quad \text{where } k = 1.0 \quad . \quad . \quad . \quad (1)$$

S_w = snow water equivalent (mm)
 S_f = snowfall depth (cm)

although, as indicated by Goodison, Ferguson, and McKay (1981), this can give very misleading snowfall data. The deficiencies in this

approach become more marked in regions that experience winter temperatures that range from well above to well below the freezing point, such as the Midwest. Typical values of k in Eq. 1 for different temperatures in the Midwest are listed in Table 4-7 below.

Table 4-7. Typical values of the empirical coefficient linking S_w and S_f at various temperatures.

Temp.(C)	k	Temp.(C)	k
-20	0.3	-5	0.8
-15	0.4	0	1.2
-10	0.6	5	2.1

This relationship also influences snowfall values measured at varying altitudes, although this is complicated by changes in precipitation patterns with altitude (e.g., Dingman, Henry, and Hendrick, 1979). Further details of the physics of changes in ice crystal structure, the properties of snow, and the relationships between snow and climate can be obtained from Mellor (1964), Hobbs (1974), Berry (1981) and Langham (1981).

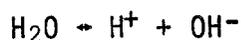
2. Airflow over/around gages: Snowfall measured at a gage may vary greatly from that actually received at that location, regardless of gage type. The total error, which usually causes an underestimate, is a combination of gage, siting, and observer errors. The gage error results from the effects of turbulence in and around the orifice, (partial) capping of the orifice, poor leveling of the orifice, catch retention due to adhesion, and losses due to evaporation/sublimation. The capping, leveling, and evaporation error components can be minimized by regular and frequent servicing of the gages, while the retention bias can be removed by the addition of a small constant (e.g., Goodison, 1978). The problems introduced by turbulence are discussed by Goodison et al. (1981, 1983). Following wind tunnel studies, it was shown that a considerable concentration of streamlines occurs immediately over the orifice, with increases in wind speed of up to 20% even with the most efficient shielding. However, one of the most effective ways of reducing these turbulence-induced errors is to provide shielding and/or nearby windbreaks (Larson and Peck, 1974; Goodison, Ferguson, and McKay, 1981). Additional correction factors, based upon wind speed at orifice height can be applied to provide more appropriate values (e.g., U.S. Army Corps of Engin., 1956; Goodison, Ferguson, and McKay, 1981).

Precipitation Chemistry: pH

Definition: $\text{pH} = -\log[\text{H}^+]$.

Note: H^+ is really H_3O^+ , where [] indicates molar concentration; M.

Dissociation of water:

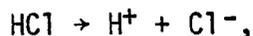


$$K_w = [\text{H}^+] \cdot [\text{OH}] = 10^{-14},$$

thus the pH of pure water $\equiv 7.0$.

Changes in pH are caused by

a) Strong acids, e.g., HCl,



if $[\text{HCl}] = 0.1$, then $\text{pH} = 1.0$.

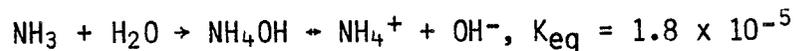
b) Strong bases, e.g., NaOH,



if $[\text{NaOH}] = 0.01\text{M}$ $\text{pH} = 12.0$ ($\text{pH} = 14 - \text{pOH}$).

Note: If the above strong acid and strong base are mixed, the result is $[\text{H}^+] = 0.1 - 0.01 = 0.09\text{M}$ or $\text{pH} = 1.05$, slightly less acid.

c) Weak acids or bases, e.g., NH_3 ,



$$[\text{H}^+] = \frac{K_w[\text{NH}_4^+]}{K_{\text{eq}}[\text{NH}_4\text{OH}]},$$

if $[\text{NH}_4\text{OH}] = 10^{-2}\text{M}$ then $\text{pH} = 10.6$.

Definition: Buffer = A complex mixture of strong and weak acids or bases and their salts, capable of neutralizing (within limits) both acids and bases to maintain an almost constant pH.

4.6.3 DESCRIPTION AND CALIBRATION OF PRECIPITATION INSTRUMENTATION

Precipitation Gages

The first rain gages were used over 2,000 years ago, and since that time a wide variety of devices of differing sophistication have been developed. However, despite the many systems employed, relatively little improvement has been achieved in their accuracy of measurement of point precipitation. In their simplest form, gages are upright hollow cylinders, open at the top end. Funnels are often added to increase the area of catch, and to reduce evaporative and retention losses. In general, gages for liquid precipitation are best with a funnel, while those for solid precipitation have an open top and are equipped with a heater or antifreeze. In the case of mixed precipitation, it is best to use a collector with no funnel, heater, or antifreeze. WMO (1973) provides an extensive annotated bibliography of precipitation measuring instrumentation. The general gage characteristics and exposure requirements necessary for the appropriate measurement of point precipitation values, and the typical sources of error in the measurements are outlined in WMO (1971). Extrapolation of point precipitation data to areal information is discussed in Bruce and Clark (1966), Barry (1969), and McKay (1970).

1. Liquid precipitation.

2. Solid precipitation: Snowfall measurements are more difficult to make and subject to greater error than those of liquid precipitation. In general, snow measurement techniques can be considered to be of two types--point or areal. The various major techniques adopted are outlined below, along with some of their individual advantages and disadvantages. If more details are desired consult McKay (1970), Teweles and Giraytys (1970), WMO (1971), Goodison, Ferguson, and McKay (1981), or Meteorological Office (1981). If the instrumentation is to be utilized in an ecosystem study, Miller (1977) provides further pertinent information.

(a) Point measurements:

Gages are used to measure Sf and Sw, while non-gage techniques are used mainly to measure Sc and Sf.

Non-gage techniques

Graduated rule/scale (Sc)

Use average of several measurements of snowdepth at representative points.

- Easy, quick, cheap, simple.
- Very subjective, difficult to detect ground surface affects area for future measurements, influenced by ice layers.

Graduated snowstake (Sc)

Permanently positioned stake positioned before winter.

- Calibrated base reference, don't have to disturb snowcover (observe from distance), quick, simple.
- Difficult to get good resolution of height, snow often collects around stake.

Snowboards (Sf, Sw)

White board set level and flush with top of snowpack. Measure snow with rule (Sc) or core snow to board (Sw).

- Separates last snowfall, simple.
- Care necessary in setup, board must have similar thermal properties to snow and be waterproof, affected by drifting.

Snow coring with sampling tubes (Sw)

Special snow-coring tubes with cutting edge; Mt Rose sampler for deep packs; Adirondack sampler for shallow packs. Core is weighed on site or in lab (bagged).

- Can be used in many types of locations, standard.
- Problems retaining core in sampler if snow is wet, ice lenses prevent good sampling, tedious in alpine environments.

Permanent reference levels (Sc)

Either a reference board placed on ground surface before winter (used with a graduated rule), or a thin steel wire strained horizontally above the snow surface between two stakes (measure down to snow surface).

- Measurements taken at same point, simple.
- Snow surface becomes disturbed.

Gages

Because of the inherent difficulties in making quality snowfall measurements, careful consideration must be given to the design and siting of gages. In general, the orifice of the gage should be carefully levelled, be at a height of at least 1-2 m above the snow surface, and have an area of cross-section of at least 200 cm² (preferably 500-1000 cm²) to reduce the potential of serious catch reductions, due to capping and restriction of the orifice by snow and ice. Additionally, the collector should have a depth of at least three times the orifice diameter to hold the catch during heavy snowstorms. The gage should also be adequately shielded from excessive turbulence around and above the orifice, either by the installation of a special shield or by siting it within a clearing in vegetation. If the later

technique is adopted, however, it must be ensured that the vegetation is not interfering with the snowfall itself. Gages are subject to retention losses due to the adhesion of water to the sides of the collector, and in the case of the Canadian collector, this has been evaluated as $0.15 + 0.02$ mm (Goodison, Ferguson, and McKay, 1981).

Non-recording gages:

Limited capacity gages (Sw)

Open-topped cylindrical collector (e.g., Canadian gage, 12.7 cm diam., 52 cm long).

- Overcome problems of sampling snowpack.
- Must be serviced regularly and frequently (daily), not suitable for remote areas, orifice is small, catch volume is limited, prone to capping.

Large capacity gages (Sw)

Up to 2540 mm Sw capacity (e.g., Sacramento storage gage). Must be positioned high to remain above pack and contain antifreeze. Melted catch is read with a dip-stick, or by telemetry from stilling well with float gage.

- Ideal for remote and/or difficult to access sites, can be left unattended for long periods.
- Must have good mixing of antifreeze to avoid a slush layer, which will freeze (use nitrogen bubbles), evaporative losses (use retardant such as oil).

Recording/totalizing gages:

Weighing type gages (Sw)

Precipitation is collected in a bucket poised on a balance, and a continuous record of precipitation (by weight) is recorded on a chart or telemetered from a strain gage. Typical types are the Belfort (Universal) and Fischer and Porter gages.

- Ideal at unattended sites, provide continuous data
- Need antifreeze, subject to capping, induce greater turbulence above the orifice because of their bulkier bodies, reduced resolution in some cases, expensive.

Shields for snow gages:

Because of the buoyant nature of falling snow, turbulence around gages must be minimized. A variety of shields have been developed to serve this purpose. Top of shield should be at or slightly above the rim of the orifice.

General purpose shields (e.g., snow fencing)

This can be used on its own or in conjunction with special shields (e.g., Wyoming shield), and may be used with an upright or slanted orientation.

- Relatively cheap and flexible in use.
- Not designed for this purpose, and is not the best.

Open or lattice shields (e.g., Alter and Tretyakov).

These shields have been designed to use with snow gages, and often consist of hinged or limited-movement slats to help prevent snow bridging.

- Provide increased catch over unshielded gages.
- Still undercatch in any other than calm conditions.

Solid or full-sheet shields (e.g., MSC Nipher, SMHI).

This type of shield tends to be smaller in dimension and attached to the gage or gage stand. Generally conical in shape (e.g., Swedish SMHI). The Canadian MSC Nipher shield was based on an earlier conical version with its shape modified to be that of an inverted bell as a result of wind tunnel tests. Following extensive field tests Goodison, Ferguson, and McKay (1981) have shown this design to be by far the most effective, at least under Canadian conditions. A modified version is being developed to fit recording gages (Goodison, Turner, Metcalfe, 1983).

- Provides catch which is closest to true.
- Unsuitable in remote areas (catches snow on rim--can blow into gage later).

(b) Areal measurements:

A variety of techniques have been developed to try and provide more appropriate areal sampling of snowpack characteristics. These techniques can be either ground based or through the use of remote sensing.

Ground-based techniques

Snow pillows (Sw)

A large air mattress filled with an antifreeze liquid, laid on the ground before snow accumulation commences and fitted with a pressure transducer that responds to variations in pressure as the snowpack changes.

- Non-destructive sampling.
- Bulky and difficult to install and maintain, response to increased weight of snow is delayed, ice layers and air gaps cause bridging errors, temperature sensitive with shallow packs, and samples only small area.

Radioisotope snow gages (Sw)

Artificial radioisotopes such as Cobalt-60 or Cesium are placed on the ground before winter, with a detector positioned above it. During snow accumulation the attenuation of the radiation is monitored and related to water content and density of the pack.

- Non-destructive sampling.
- Only small area sampled, radiation hazards, licensing needs, expensive.

Natural gamma radiation (Sw)

Use a portable gamma-ray spectrometer to determine pre-snow and during snow gamma levels and then relate these values to snowpack water content.

- Non-destructive sampling, no radiation hazard.
- Only useful for shallow packs, sensitive to instrument drifts, affected by changes in soil moisture and radon gas washout.

Snow Surveying (Sw and Sc)

Take snow cores at a series of premarked locations (snowcourse) and combine data to get an area-wide index of snow water equivalent. Use Mt. Rose/Federal sampler for deep snowpacks, and Adirondack sampler for shallow snowpacks. Needs careful preselection of sampling sites (e.g., WMO, 1974).

- Provides opportunity to get good areal sampling of basins, provides long-term network data.
- Tedious and time-consuming task, subjective selection of sampling points, problems of snow coring tubes, and destructive sampling.

Remote sensing techniques

Aerial Surveys of Snow Markers (Sc)

Observations of snow depth at snow markers and of snowline elevation from aircraft.

- Well suited to mountainous and remote locations that are difficult and expensive to access.
- Very expensive and time-consuming, resolution is low, safety problems, and can only be made in suitable weather.

Natural Gamma Radiation by Aerial Survey (Sw)

Monitor natural gamma radiation emitted from the earth using airborne sensors. This radiation is attenuated by water between the surface and the sensor. Relate data to snowpack water equivalent, using previous and during snowcover values.

- Gives good areal coverage--mean Sw value for area, good for inaccessible locations.
- Sensitive to water in air between aircraft and ground, and to changes in soil moisture and background radiation, needs precision navigation to ensure previous and during snowcover flight tracks are the same.

Microwave Sensing from Aircraft (Sw)

Experimental studies of the use of both radar and passive microwave systems (0.1-100 cm) for the determination of snowcover characteristics and the onset of snowmelt are being undertaken.

- Capable of sensing through cloud cover, capable of high resolution.
- Expensive, not yet operational.

Satellite Observations (Sc)

Useful data are provided by the polar-orbiting Landsat, NOAA satellites, and by the geostationary SMS/GOES satellite.

- Wide areal coverage--good for large areas, data telemetered.
- Viewing angles can cause distortion, resolution variable, and temporal resolution not always adequate.

Precipitation Chemistry: pH Measurement

1. Dyes - There are many different dyes that take on a distinctive color at a specific pH but these dyes only offer discontinuous analysis. Most dye paper is accurate only to +1 pH unit. Special dye paper in the range of pH 4 to 7 is more accurate.

2. Titrations - offer very accurate means of determining total acidity or alkalinity and thus pH, but such analyses are time consuming.

3. pH Electrodes - the usual method of pH measurement. They are quick, reasonably accurate, and inexpensive:

Response to $[H^+]$.

Nernst Equation relates concentration to potential:

$$V = V^0 + \frac{RT}{nF} \log \left(\frac{A_{ox}}{A_{red}} \right) .$$

V = volts out
 V^0 = potential of reference
 R = gas const $8.314 \text{ j k}^{-1} \text{ mole}^{-1}$
 n = number of electrons (here $n = 1$)
 F = Faraday = 96500 Coulombs
 T = Temp in K
 A_{ox} = Molar conc. of oxidized form
 A_{red} = Molar conc. of reduced form.

Note: Both pH and the potential, V, in the Nernst equation are proportional to the log of the concentration, thus pH electrodes respond linearly to pH.

Commercial glass pH electrodes generally consist of a thin glass bulb containing a solution of HCl and an internal reference electrode of silver and silver chloride. The output is compared to an external (or second internal) electrode.

For more details see:

Stroebel (1973), 660-663
 Fritz and Schrenk (1969), 337-342
 Day and Underwood (1980), 306-310.

Calibration is achieved through the use of standard buffer solutions. The electrode should be calibrated frequently--daily--or before each use. It is best to calibrate with buffers of pH above and below the expected value. For precipitation, this might be NBS buffers at pH 4.01 and 6.86. These buffers can be easily prepared (Stroebel, p. 475) or purchased commercially.

Errors are caused by a slight sensitivity to Na^+ and by the removal of H_2O from the glass by strong acids.

Uncertainty: For most applications a pH measurement should not be considered more accurate than +0.05 pH units.

Other ions: There are wet chemical/spectroscopic and specific ion electrode methods of analysis of the cations and anions commonly present in precipitation. These are described in the reference, but are too involved for a course on general meteorological instrumentation. The newest, fastest, and most convenient method of analyzing the various ions (such as SO_4 , NO_3^- , Cl^- , Na^+ , Ca^{++} , Mg^{++}) is ion chromatography. Sawicki, Mulik, and Wittgenstein (1978) offer a good review of the techniques. An ion chromatograph costs in the neighborhood of \$20,000.

4.6.4 APPLICATIONS

Selection of Instrumentation and Siting

1. Funds/equipment availability.

2. Nature of site.

- site accessibility/remoteness
 - remote or observer tended
- site exposure and exposure errors
 - terrain
 - vegetation
 - other obstructions
- known precipitation characteristics of site
 - nature of precipitation expected--solid?
 - expected rates/amounts

3. Gage comparisons--accuracy.

Sampling, Processing and Recording

1. Liquid precipitation.

- double mass curve analysis

2. Solid precipitation.

Sampling rates for solid precipitation vary with the purpose of the measurement site, its accessibility, and its staffing. Stations, which form part of a much larger network (e.g., NWS sites), usually monitor S_f and/or S_w with point measurement techniques, either on a daily or continuous basis. Lower-order stations monitor S_f , while higher-order stations monitor S_w and sometimes S_f as well. The Canadian AES network uses the MSC Nipher-shielded gage as its official instrument to monitor S_w , but at the moment only about 370 of its 2500 precipitation stations are equipped with them. The other stations measure S_f using a graduated rule and convert to S_w estimates using Eq. 1 with $k = 1.0$. The Nipher gage is used only to measure snowfall, with a standard rain gage being used to monitor liquid precipitation. The NWS network of the USA, along with other countries such as Sweden and the USSR, use the one gage for all precipitation measurements. In cases of watershed studies that monitor the snowpack for the prediction of water during snowmelt to assess flooding potential, usually S_c and S_w are measured on a less frequent basis (e.g., weekly or biweekly). Also, areal techniques are adopted in the main. Once obtained, the data are either converted to S_w values or used directly in predetermined empirical relationships with other factors of interest, such as flood potential. Analysis of S_w data can utilize the same techniques as for liquid precipitation, including such statistical methods as extreme value frequency analysis, double mass curve analysis, isohyetal analysis, and Thiessen polygon analysis, in addition to standard central tendency and variance statistics (e.g., Bruce and Clark, 1966; McKay, 1970; WMO, 1974; and Haan, 1977). Goodison, Ferguson, and McKay (1981) provide a good description of fundamental analysis and data presentation techniques.

3. Precipitation chemistry

- a) Most pH electrodes require at least 10 cm³ of sample.
- b) Samples should be analyzed as soon as possible after collection, or kept refrigerated or spiked with chloroform.
- c) More information about the nature of the atmosphere can be obtained if precipitation is sampled sequentially throughout an event, but such systems are difficult to automate.
- d) More information can be obtained if each event is sampled individually. For example, monthly precipitation samples would almost certainly have changed pH between event and analysis.
- e) Wet only sampling is preferred because dust can collect in the samples, and alter the observed pH. Wet and dry sampling is not even a good indication of dry deposition because the sticking coefficient of the sampler, is not the same as the soil or vegetation.
- f) Cannot site gauge at ground level due to dust.
- g) pH is independent of precipitation amount, but total acid deposition in an area is not.

Networks and Spatial Extrapolation of Data

Usually precipitation data are collected to provide an areal estimation of the atmospheric water input to the surface of the earth. However, since these data are mainly obtained from point measurement systems, it is necessary to use either a network of monitoring sites and/or an extrapolation technique to obtain the required areal estimates.

1. Networks

Precipitation data collection networks, usually consisting of sites equipped with point measurement systems and of varying areal extents and sampling densities, have been established for a variety of purposes. The individual network stations can be permanently staffed, visited on a routine basis, or automated (e.g., refer to examples in WMO, 1975).

- Purpose: Some purposes for the establishment of precipitation networks are to provide data for use in forecasting and for archiving and future use in climatological or post-synoptic studies, for special purposes, such as flood forecasting, and for special research studies designed to improve our understanding of the precipitation process. The following are some examples of these networks.

- National Weather Service Network:
Nationwide network of recording and stick gages that provide data for use in forecasting and for archiving.
- Watershed Networks:
Limited area networks with data provided for a specific purpose, such as flood forecasting (e.g., Sleepers River watershed, VT: see Anderson et al., 1977). Often automated, usually not permanently manned.
- Special Research Networks:
Usually related in size and density of monitoring sites to the systems under investigation. In Illinois several precipitation networks of this type (e.g., VIN, METROMEX, CHAP) have been established, with time resolutions of from 5-60 min, and space resolutions as small as 0.3 km (Changnon, 1977).
- Representative sampling: One of the most critical decisions in the development of precipitation networks required to provide a high-quality spatial representation is the selection of measurement site locations. The locations selected must provide a proportionate representation of the different exposures that will have varied influences on the pattern of incoming precipitation.

2. Spatial extrapolation methods

- arithmetic mean
- Thiessen polygon
- depth-area-duration analysis

4.6.5 QUESTIONS, PROBLEMS, AND LABORATORY EXERCISES

Questions

1. Under what conditions would you prefer to have snow precipitation data: in the form of snowfall, snowfall water equivalent, or snowcover measurements?
2. Compare the advantages and disadvantages of using limited capacity, storage, and recording snow gages. Under what conditions is each type preferable for field installation?
3. Discuss the problems involved in:
 - establishing representative snow gaging points with appropriate instrumentation, and

- determining areal estimates of winter precipitation from gage measurements for both mountainous watersheds and flat prairies.
4. Outline the advantages and disadvantages of determining areal estimates of snow cover by remote sensing techniques using aircraft and satellites.
 5. Discuss the relative merits of the various types of shielding that can be used to reduce the effects of turbulence around the orifice of snow gages. Why might the MSC Nipher shield cause a slight overcatch at wind speeds of about 0.5-3.0 m/s? When would a Wyoming shield be preferable?
 6. Discuss the practical, technical, and measurement problems associated with snowcoursing in a mountainous watershed and a subarctic area. In view of these difficulties, why is this technique of snow surveying utilized so widely?
 7. Discuss the problems that would confront you if you had to supervise the establishment of a network of snow measuring devices or some other form of areal sampling of the winter snowcover for:
 - a large mountain watershed in the Sierra Nevada for the purpose of advance warning of flood potential and to be funded by the state government,
 - a small watershed that supplies a local water supply funded by county revenue, and
 - a national network with the data to be used for forecasting and archival purposes.

Questions and Answers

1. Q. Given mean pH values for a number of events, what must be considered when calculating the mean pH for all the events?
 - A. pH is a log; therefore, it's best to convert to $[H^+]$ before taking the mean, and then recalculating the pH. Don't forget that the samples should be volume weighted.
2. Q. What are the effects on precipitation pH of the following: Power plant SO_2 , auto emission NO_x , fertilizer NH_3 , wind blown dust containing $Ca(OH)_2$, sea salt aerosol?
 - A. $SO_2 \rightarrow H_2SO_4$ acid, lower pH
 $NO_x \rightarrow HNO_3$ acid, lower pH
 $NH_3 \rightarrow NH_4OH$ base, higher pH
 $Ca(OH)_2 \rightarrow Ca^{++} + OH^-$ base, higher pH
 $NaCl \rightarrow Na^+ + Cl^-$ no effect

3. Q. Why does the pH of a sample of precipitation change with time?

A. SO_4^- , NO_3^- , and NH_3 are consumed by microorganisms.

Problems

1. The following data were measured at the Urbana, IL weather station on the dates listed. Use the snow density vs temperature relationship in conjunction with the Sw data to check the Sf values and the observed change in Sc. Account for any apparent discrepancies.

Date	Temp. (C)	Sw (mm)	Sf (cm)	Change in Sc (cm) *
Feb 4, 1978	-9.0	0.7	1.1	0.6
Feb 13, 1978	-3.3	23.9	27.7	16.8
Feb 28, 1978	-1.7	3.3	3.6	2.3
Apr 14, 1980	1.8	15.6	11.7	2.5
Nov 27, 1980	-0.6	31.0	27.7	20.3
Dec 16, 1981	-5.0	14.5	18.0	17.8
Dec 22, 1981	0.0	17.0	15.0	10.2
Dec 28, 1981	-6.7	9.4	10.2	7.6

* Taken at 7a.m. CST on day after end of snow storm.

Comment on the usefulness of this relationship and cautions that should be considered. Would any other data be useful in your analysis? Explain.

2. The following table lists Sw data from 3 snowcourses sited in the Sleepers River watershed, VT. Each set of data presented consists of averages of 10 measurements made on each snowcourse during the same day. Data for a similar day for each year from 1960-1974 have been selected, using a date in March close to the time of snowpack maximum. Snowcourse R-1 was in a clearing about 20 meters across in an area of 10-15 meter high spruce, R-3 was situated adjacent to and south of an area of mixed forest of 10-15 meter height, while R-6 was located in an open field north of an area of dense spruce and south of a small road and further open field. Examine the snowcourse data presented, and using the most appropriate analytical techniques, determine their integrity. Have you any recommendations for amendments to the snowcourse sites or rejection of some data? (Data taken from Anderson et al., 1977).

Date	Average Sw values (mm) from snowcourses		
	R-1	R-3	R-6
March 20, 1960	287	274	165
March 20, 1961	163	165	145
March 19, 1962	241	241	182
March 21, 1963	399	371	396
March 2, 1964	182	165	231
March 5, 1965	130	117	81
March 14, 1966	274	282	180
March 27, 1967	142	124	226
March 19, 1968	142	183	386
March 28, 1969	352	354	610

3. See Day and Underwood (1980), pp 322-325
 Fritz and Schrenk (1969), pp 162-165 and 197-200
 Stroebel (1973), pp 684-686

Laboratory Exercises

Laboratory Exercise #1

This field (laboratory) exercise requires cooperative weather conditions and is best suited to the January-March period and to the northern or mountainous states. It can be performed as a class demonstration or student exercise.

Objective:

Comparison of different methods of measurement of Sf and Sw, including the use of variously shielded gages.

Equipment:

The degree of sophistication of the equipment used can be varied widely to suit the resources available.

- (1) Snow measurements:
 Gages (at least two of same type), shields (including sections of snow fence and Alter shield--if available), snow-coring tube, graduated rule, two graduated stakes, at least two labelled stakes.
- (2) Supplementary meteorological measurements:
 Anemometer, temperature sensor, recording system if necessary
- (3) Other equipment:
 Balance (or access to) capable of weighing to 5 kg, plastic bags and ties.

- (4) Measurement sites:
At least two areas (A,B,...) where the snowcover will not be disturbed, one of which (A) is in a well-sheltered (from wind) location, and the other (B) in a much more open site.
- (5) Observer(s):
One or more well-humored observers (or students)!

Method:

The array of equipment at each site and its layout will depend largely upon equipment availability and the nature of the site.

- (1) Site A will provide the "truth" measurements, while site B will provide data from measurement points influenced by wind and turbulence effects.
- (2) At site A set up a gage in a position where it will be influenced by little wind and, if necessary, use additional shielding (snow fence). Install a snow stake (graduated) and labelled stake(s) (for snow-core locations) nearby.
- (3) At site B set up a similar gage (unshielded), snow-stake, and labelled stake(s) to those installed at site A. If additional gages are available, install them at site B and provide them with a variety of shielding. Some types of shielding that could be considered are
 - snow fencing - vertically or obliquely supported
 - single or double row of fencing
 - Alter shield - with or without additional shielding
 - solid shield - such as a sheet of metal around gage in the shape of an inverted cone.

Make sure that the top of the shielding is not above the gage orifice, and that the gage orifice is level.

- (4) Take measurements of S_f and S_w at least before and after major events.
 - S_f - graduated rule, graduated snow-stake
 - S_w - gages; snowcoring at labelled stake(s) - take at least three cores, secure them in labelled, pre-weighed plastic bags, and return them to the laboratory for weighing and determining S_w

Analysis: The analysis will vary according to the equipment used and the sampling frequency. Sf values should be converted to estimates of Sw using Eq. 1 with values of k set to 1 and according to the relationship shown in Table 4-7. Ratios of gage catches at site B to those at site A should be plotted against wind speed for each of the gage types used. Other comparative statistics and analytical techniques could be employed.

NOTE: Accumulation of data of this type over a number of winter seasons will provide a much better data base for these analyses, and will permit the lab exercise to become an indoor event if this is preferred.

Laboratory Exercise #2

See Stroebel (1960), p. 630
Daniels et al. (1970), pp 186-191

It also is suggested to have students test effect of adding soil or gravel (about 1g/100ml) to distilled water, tap water, rain water, sea water, and a buffer solution.

4.7 EVAPORATION

4.7.1 REQUIRED BACKGROUND

- Elementary thermodynamics, particularly: Dalton's Law
Clausius-Clapeyron Equation, humidity variables.
- Elementary micrometeorology.

4.7.2 THEORETICAL BACKGROUND

Evaporation occurs if the net exchange of water vapor molecules at the interface of liquid water and air is upwards. This exchange depends on the difference in water vapor pressure at the surface, $e(o)$, and in the air above, $e(a)$, and on the removal mechanism, which is a function of the wind speed, u . Accordingly, an estimate of evaporation can be obtained by applying one or more of the following principles.

Mass Transfer or Bulk Aerodynamic Method

Dalton expressed the relationship between evaporation, water vapor deficit, and windspeed in 1802 as

$$E = b f(u) (e_o - e_a).$$

A weak dependence on total atmospheric pressure is usually neglected. The proportionality coefficient b depends on the height z of measurement of u and $e(a)$ and on turbulence characteristics of the flow (stability regime), as well as on surface properties (roughness, permeability, soil moisture, and vegetation).

Mass Budget

The mass deficit of a body of water or a block of soil is either directly measured or calculated from the mass budget equation

$$E = I + P - O + St,$$

where I = Inflow
 O = Outflow
 P = Precipitation
 St = Storage

If I and O are zero, as in pans and evaporimeters, the change in water depth, corrected for eventual precipitation fallen, represents evaporation. In Lysimeters the water loss of a vegetated block of soil is determined by gravimetric methods. Larger bodies of water, such as lakes and reservoirs, can be used if accurate measurements of inflow

and outflow, as well as of precipitation over the water surface can be made.

Energy Budget

Because energy is necessary for evaporation, a measurement of all other terms in the energy balance of the evaporating surface can be made to determine evaporation from the latent heat flux Q_e . The energy budget can be expressed as

$$Q_{St} = Q_s(1 - A) - Q_b - Q_h - Q_e + Q_{ad}$$

where:

- Q_{St} = rate of heat storage below the surface
- Q_s = solar radiation incident on the surface
- A = albedo of surface
- Q_b = net long-wave radiative flux
- Q_h = sensible heat flux
- Q_e = flux of latent heat
- Q_{ad} = heat advected into unit volume below surface (only if the surface is water)
- Q_g = conductive flux of heat from deeper layers in the ground or water.

Minor terms in the heat balance, such as the heat of precipitation falling onto the surface, or kinetic heat dissipated in breaking waves or other motions, or chemical-biological energy exchanges, are neglected.

The ratio of sensible heat flux to the latent heat flux, called the Bowen ratio, can be independently determined from the respective gradients

$$R_b = Q_h/Q_e = \gamma \frac{T_s - T_a}{e_s - e_a} \frac{p}{1000} .$$

The psychrometric constant $\gamma \approx 0.61$, if T_s and T_a are measured in degrees C, e_s and e_a and p in mb. The subscripts a and s refer to the air and the surface, respectively.

$Q_n = Q_s(1 - A) - Q_b$ is the net radiation, measurable with one instrument or calculated.

The evaporation can then be expressed through the energy balance and the Bowen ratio as

$$E = \frac{Q_n + Q_{ad} + Q_{St}}{\rho L (1 - R_b)} .$$

Combined Mass Transfer and Energy Balance (Penman)

Penman combined the mass transfer (Dalton type equation) with the energy balance to yield an empirical evaporation formula, which has found wide acceptance

$$E = \frac{\Delta R_n + \gamma E_a}{\gamma + \Delta}$$

where γ , the psychrometric constant, and Δ , the slope of the saturation water vapor vs temperature curve at the surface temperature, are treated as weighting factors. Penman's equation can be further modified to include climatic and biological parameters to yield monthly values of evapotranspiration.

Eddy Flux

Turbulent motions in the air above the surface will be more effective in removing moist air from near the surface to greater heights than molecular motions (diffusion). Over a large homogeneous surface and under steady state conditions, this transport by eddy motions should be constant with height. Within this constant flux layer (Prandtl layer) the vertical transport of water vapor, which is equal to the evaporation at the surface, can be measured by the covariance of the vertical component of motion, w , and the water vapor concentration, q , (or mixing ratio m)

$$E = \rho \langle wq \rangle,$$

where $\langle \rangle$ denotes a time average. An instrument system capable of measuring this covariance, called an eddy flux system or evapotron (a name given to an early Australian system), will typically consist of sensors for the three components of motion and of water vapor concentration (and usually also of temperature), as well as of the digital or analog data processing of the sensor outputs to perform the multiplication and time integration of the product wq .

Gradient Flux

The flux of water vapor E , under steady state and homogeneous surface conditions, can be formally expressed by an eddy exchange coefficient or "eddy-diffusivity," K_E , multiplied by the mean vertical gradient of water vapor concentration q

$$E = \rho K_E dq/d\bar{z} .$$

Under near neutral stability conditions and/or close to the surface, it can be assumed that the exchange mechanism for water vapor is the same as for momentum, and therefore, that the ratio of the eddy-diffusivity and the eddy-viscosity, K_E/K_M , is nearly one. The

water vapor flux, or the evaporation E , can then be measured by the ratio of the gradients, respectively of the finite differences in water vapor concentration and of the wind speed (u -component) at two levels relatively close to the surface, e.g., 1 m and 3 m. This rationale resulted in a number of practically applicable evaporation formulae of which those due to Thornthwaite-Holzman

$$E = k^2 \rho \frac{(q_1 - q_2) (u_1 - u_2)}{[\ln(z_2/z)]^2}$$

modified by Pasquill for measurements very close to the surface

$$E = \frac{k^2 [1 - (u_1/u_2)]}{[\ln(z_2/z_1)]^2} \rho u_2 (q_1 - q_2)$$

and due to Sverdrup

$$E_L = \frac{\rho k u_* (q_s - q)}{\ln [(z+z_0)/(\delta_1+z_0)] + (k\delta_1 u_*)/D}$$

are most widely used.

In the above formula, δ_1 is the thickness of the laminary boundary layer, z_0 is the roughness length, u_* is the friction velocity, and D is the coefficient of molecular diffusion of water vapor in air.

For measurement levels of 2 m and 8 m, the last two equations are written by Thornthwaite-Holzman as

$$E = \frac{6.2 k^2 (u_8 - u_2) (e_2 - e_8)}{p (\ln 800/200)^2}$$

and by Sverdrup

$$E_L = \frac{0.62 \rho k^2 u_8 (e_0 - e_8)}{p (\ln 800/z(0))^2} .$$

4.7.3 NATURE OF VARIABLES AND PURPOSE OF MEASUREMENTS

Five different amounts of evaporation are defined, according to the method of measurement:

E_a = bulk aerodynamic evaporation (mm/day)
 Q_E = LE, latent heat flux density (Watt/m²)
 E_t = evapotranspiration (mm/day)
 E_L = lake evaporation (mm/day)
 E_p = pan evaporation (mm/day).

Evaporation is, strictly speaking, the flux of water mass per unit time and unit surface area. Since the density of water is nearly one in international units, mass can be expressed as depth per unit surface area, and day is used as unit of time; hence, mm/day is an international unit for evaporation. For studies over large areas, volumes are often expressed in acre-ft.

For energy budget considerations, $Q = LE$, the latent heat flux rather than the mass flux, is the relevant quantity. L is a weak function of temperature and amounts to 2.5 kJ/g at 0°C.

Numerical values in the above evaporation equations depend not only on the units of measurement, but also on the height above ground.

The amount of water evaporated is needed in three types of problems:

Agriculture: The consumptive use of plants has to be determined, i.e., transpiration or evapotranspiration, which is highly dependent on the type of plant cover, soil moisture, and groundwater.

Water resources management: The water loss from lakes and reservoirs, i.e., free water evaporation, is needed.

Energy budget considerations: A knowledge of the latent heat flux is important in many energy related problems, from cooling towers to energy conservation.

4.7.4 DIRECT MEASUREMENT DEVICES

Atmometers

- Piche
- Bellani
- Alundum

Evaporation Pans

Evaporimeters

Lysimeters

Eddy Flux (evaporitron)

Gradient Flux (wind, temperature, and humidity profilers).

Direct measurement devices are adequately described in most text books on meteorological instruments; see WMO (1971) Guide. Their main limitation is that they measure "potential evaporation," which depends very critically on size and exposure and does not indicate actual evaporation occurring from natural surfaces. This comment, however, does not apply to Lysimeters and systems nor to direct measurement systems, eddy flux and gradient flux. The sensor requirements, as can be seen from the defining equations, are as follows:

Eddy flux measurements: Sensors of short response times and high sensitivity are needed. The response characteristics of both sensors have to be well matched. If the evaporative flux is to be measured at an elevation below about 2 m and/or stable to neutral conditions, response times well below 1 s are necessary. The non-dimensional response time (following Priestley (1965)) is

$$\tau = 2.5 t_i u/z .$$

This relationship stems from a comparison of the amplitude reduction factors of an instrument with time constant t_i and a sinusoidal function subjected to smoothing by a running average interval t_m , where t_i is the instrument's response time. If $\tau = 1$ for both sensors, about 80% of the evaporative flux is recorded under inversion conditions, and more than 90% are recorded under lapse conditions, see Priestley (1965). Also, compare sections on wind, temperature, and humidity sensors.

Gradient flux measurements: Mean profiles or difference of humidity and wind speeds at two or more levels are needed. Sensors have to be well matched (good relative accuracy or precision) and have high sensitivity and long response time.

Indirect Methods

- Mass Transfer (bulk aerodynamic)
- Energy Budget
- Water Budget
- Hybrid (Penman)

4.7.5 QUESTIONS AND LABORATORY EXERCISE

Questions

What is an appropriate method to measure:

- The actual short time (one hour) evaporation from any natural surface?

- The monthly evaporation from an extended natural surface such as a watershed?
- The "potential evaporation" at a given location?
- The lake evaporation from a given lake or reservoir?

Laboratory Exercise

A small dish of water is exposed to a constant airstream from a fan or in a wind tunnel. Dry- and wet-bulb temperatures of the airstream upwind and downwind of the dish, as well as water temperature in the dish are continuously recorded. Wet- and dry-bulb temperatures in the air will show little change, but the water temperature in the dish will reach a new equilibrium temperature noticeably below the wet bulb and even below the dew-point temperature of the air.

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